

Alabama School of Fine Arts
Invitational Mathematics Tournament

January 7, 2006

Algebra I Exam

Directions:




1. Make sure your name and student number are bubbled correctly on the pink answer sheet.
2. No books, notes, calculators, or other aids may be used. Scratch paper will be provided by the exam proctor.
3. You may write on this exam booklet; however, all answers must be recorded in the proper places on the pink answer sheet. The pink answer sheet must be given to the exam proctor when time is called.
4. All answers must be simplified. Do not round unless stated in the question. Units are not required in an answer. If a certain form for the answer is requested, be sure to use that form.
5. This exam consists of 25 multiple choice questions with A, B, C, D, and E as answer choices. There are three tie-breaker questions: TB1, TB2, and TB3. Write all answers to tie-breaker questions on the back of the pink answer sheet, labeled with the respective number.
6. “NOTA” denotes “None of the above.”
7. Each correct answer earns 4 points. For each incorrect answer, 1 point is subtracted. There is no penalty for unanswered questions.

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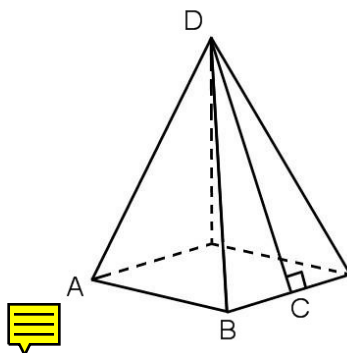
- 1) If $a = bc$, which of the following must be equal to ab ?
- A. bc
 - B. bc^2
 - C. b^2c
 - D. $\frac{b}{c}$
 - E. NOTA
- 2)  Nine politicians had just finished making a dirty deal and they wanted to congratulate each other. They began shaking hands, but each politician only shook hands with every other co-conspirator just once. How many times did the politicians shake hands?
- A. 9
 - B. 36
 - C. 81
 - D. 100
 - E. NOTA
- 3)  SFA, people are classified as Wigs, Wags, and Zags. In one particular class of 37, there are 23 Wigs, 18 Wags, and 31 Zags. Also, 11 people are both Wigs and Wags, 19 people are both Wigs and Zags, and 14 people are both Wags and Zags. How many people are Wigs, Wags, **and** Zags?
- A. 5
 - B. 7
 - C. 9
 - D. 11
 - E. NOTA
- 4)  Subtract: $11000_2 - 1011_2$
- A. 1000_2
 - B. 1001_2
 - C. 1101_2
 - D. 1111_2
 - E. NOTA



- 5) If x is a negative integer, which of the following could be an odd integer?
- A. $2x + 2$
 - B. $x^3 - x$
 - C. $x^2 + x$
 - D. $7x - 3$
 - E. NOTA
- 6) If $r - t = t - s = s - t$, then which of the following must equal $2r - 2s$?
- A. $3r - 2t$
 - B. $4r - 4t$
 - C. $3r + 2t$
 - D. $3r + 3t$
 - E. NOTA
- 7) Let x be the smallest positive integer such that: when x is divided by 3, the remainder is 1; when x is divided by 5, the remainder is 2; when x is divided by 7, the remainder is 3. Find the remainder when x is divided by 11.
- A. 6
 - B. 7
 - C. 8
 - D. 9
 - E. NOTA
- 8) Dauntorious and Glomerulus both like red Gummie Bears. The assorted pack of bears they bought came with 10 greens, 12 reds, 8 pinks, 13 yellows, and 7 oranges. They each take turns drawing one Gummie Bear at a time, without replacement. What is the probability that Dauntorious and Glomerulus will both get a red Gummie Bear on their first draws?
- A. $\frac{6}{25}$
 - B. $\frac{66}{1225}$
 - C. $\frac{72}{1225}$
 - D. $\frac{66}{1250}$
 - E. NOTA
- 9) Find the 2006th term of the following sequence: 1, 4, 7, 10, 13, ...
- A. 2006
 - B. 6016
 - C. 6018
 - D. 6019
 - E. NOTA



10)



The pyramid in the figure above has a square base and all 4 triangular faces are congruent. If the length of \overline{AB} is 1 inch and the total surface area of the pyramid is 5 square inches, what is the length of \overline{CD} in inches?

- A. $\frac{1}{2}$
- B. $\frac{\sqrt{3}}{2}$
- C. 1
- D. 2
- E. NOTA

11) If $x < -4$ or $x > 4$, which of the following must be true?

- I. $x^2 > 4$
 - II. $|x| > 4$
 - III. $x^3 > 4$
- A. III only
 - B. I and II only
 - C. I and III only
 - D. II and III only

 NOTA

12) A jar has 4 amoebas in it. Amoebas split their cells in two (doubling in size) once every minute. The jar will be completely filled in 10 minutes. How many minutes would it take to fill the same sized jar if had 8 amoebas in it to start?

- A. 2.5
- B. 5
- C. 8
- D. 9
- E. NOTA



- 13) Gas prices are going through the roof! They are rising approximately 30% per month. At this rate, in how many months (rounded up to the nearest month) will the price be more than 4 times the current price?

($\log_{1.3} 4 = 5.284$, if that helps)

- A. 4
- B. 5
- C. 6
- D. 7
- E. NOTA



- 14) Multiply: $(100,005)(99,995)$

- A. 1,000,000,005
- B. 2,052,529,241
- C. 9,999,999,995
- D. 9,999,999,975
- E. NOTA

- 15) If $f(x) = \frac{2}{2x-3}$ and $g(x) = \frac{2+3x}{2x}$ ($x \neq 0, \frac{3}{2}$), then what is $f(f(g(f(g(f(f(2)))))))$?



- B. $\frac{4}{3}$
- C. $\frac{5}{4}$
- D. Does not exist



NOTA

- 16) $Ax^2 + Bx + C = 0$. If $A + B = 54$, $B + C = 59$, and $A + C = 37$, find the sum of all real values of x that satisfy the equation.

- A. $\frac{7}{8}$
- B. $-\frac{19}{8}$
- C. $-\frac{21}{16}$



$\frac{21}{16}$

- E. NOTA

17) What is the area of the region that satisfies all three inequalities below in the Cartesian plane?

$$y \leq 2x + 3$$

$$y \geq -x$$

$$y \leq -5x + 10$$

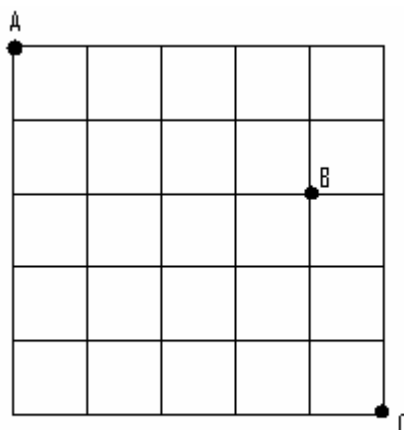
- A. 10
- B. 10.5
- C. 11
- D. 11.5

 NOTA

18) $2^{1000} + 2^{100}$ is closest to which of the following on the real number line?

- A. 2^{1000}
- B. 2^{1100}
- C. 2^{1001}
- D. 4^{1001}
- E. 4^{1002}

19)



In the figure above, how many ways are there to get to Point C from Point A, if you can only move right or down along the lines, and you must go through point B?

- A. 19
- B. 38
- C. 60
- D. 252
- E. NOTA


20) The equation $xy = 144$ has how many solutions (x, y) where x and y are positive integers?

- A. 8
- B. 12
- C. 15
- D. 16
- E. NOTA




21) In January, 72 lunches were served at the ASFA Cafeteria, for a total of $\$a67.9b$, where a and b are digits. What is $a + b$? (Hint: the total price must be divisible by 72)

- A. 2
- B. 5
- C. 8
- D. 9
- E. NOTA


22) The mean of the set $\{x, 30, 50\}$, written in no particular order, is equal to its median. Find the  of all possible x -values.

- A. 120
- B. 70
- C. 30
- D. 10
- E. NOTA

23) The sum of n consecutive integers is 2006. What is the largest possible value of n ?

-  59
- B. 68
- C. 236
- D. 1003
- E. NOTA

24) If:


 $\frac{A+3}{7+\frac{15}{A}} = \frac{1}{2}, (A > 0);$

B is the 6th triangular number;

C is the units digit of 7^{2006} ;

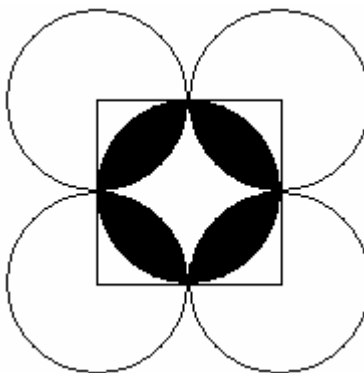
$\frac{1}{D}$ is the probability of choosing a club from a regular deck of cards

Find

 $\frac{A - B^2 + \frac{C}{A}}{CD - (B + C + 2A)}.$

- A. 5
- B. 6
- C. 7
- D. 8
- E. NOTA

25)



In the figure above, a square with sides measuring 14 cm has 4 circles with centers at the vertices of the square. The radius of these circles is 7 cm. There is also a circle inscribed inside the square. What is the area of the shaded region?

- A. $3.5\pi - 14$
- B. $7\pi - 49$
- C. $84\pi - 140$
- D. $98\pi - 196$
- E. NOTA

Tiebreakers:

TB1:

What is $1000 + 1001 + 1002 + \dots + 2005 + 2006$?

TB2:

Solve for x: $\frac{64^{x-2}}{4^{x-2}} = 256^{2x}$

TB3:

Which of the following statements is/are true (write down number(s) on answer sheet)?

1. Exactly one of these ten statements is false
2. Exactly two of these ten statements are false
3. Exactly three of these ten statements are false
4. Exactly four of these ten statements are false
5. Exactly five of these ten statements are false
6. Exactly six of these ten statements are false
7. Exactly seven of these ten statements are false
8. Exactly eight of these ten statements are false
9. Exactly nine of these ten statements are false
10. All of these ten statements are false

Answer Key - Algebra 1

1. C

2. B

3. C

4. C

5. D

6. B

7. C

8. B

 B

10. D

11. B

12. D

13. C

14. D

15. A

16. B

17. B

18. A

19. C

20. C

21. B

22. A

23. E - 4012

24. E - does not exist

25. D

TB1 1513521

TB2 $x = -\frac{2}{3}$

TB3 9