

Alabama School of Fine Arts  
Invitational Mathematics Tournament

January 6, 2007

Algebra I Exam

Directions:

1. Make sure your name and student number are bubbled correctly on the pink answer sheet.
2. No books, notes, calculators, or other aids may be used. Scratch paper will be provided by the exam proctor.
3. You may write on this exam booklet; however, all answers must be recorded in the proper places on the pink answer sheet. The pink answer sheet must be given to the exam proctor when time is called.
4. All answers must be simplified. Do not round unless stated in the question. Units are not required in an answer. If a certain form for the answer is requested, be sure to use that form.
5. This exam consists of 25 multiple choice questions with A, B, C, D, and E as answer choices. There are three tie-breaker questions: TB1, TB2, and TB3. Write all answers to tie-breaker questions on the back of the pink answer sheet, labeled with the respective number.
6. “NOTA” denotes “None of the above.”
7. Each correct answer earns 4 points. For each incorrect answer, 1 point is subtracted. There is no penalty for unanswered questions.

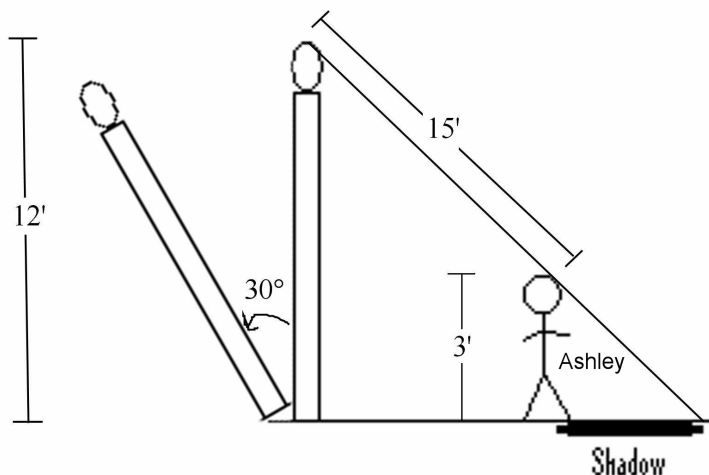




- 12) If  $x_0 = 1, x_1 = 2$  and
- $$\frac{x_{n+1}}{x_n} = \sqrt{\frac{x_n}{x_{n-1}} + 2}, \quad x_4 \text{ is}$$
- A. 16  
B. 8  
C. 24  
D. 28  
E. NOTA
- 13) If 2007! was written in base 12, how many zeros would there be if you started at the right of the number and counted zeros to the left until coming to the first non-zero digit?
- A. 998  
B. 1000  
C. 1999  
D. 1998  
E. NOTA
- 14) Simplify:  $\sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}}$
- A.  $\frac{\sqrt{2}}{2}$   
B.  $2\sqrt{2}$   
C.  $2 - \frac{\sqrt{2}}{2}$   
D. 2  
E. NOTA
- 15) In Madame Lee's class of 35 students, a student speaks French, Spanish, English, or any combination of the three. 20 students speak English, 17 speak Spanish, and 19 speak French. 8 speak both English and Spanish, 10 speak English and French, and 2 speak all three languages. How many students speak French and Spanish only?
- A. 5  
B. 7  
C. 10  
D. 15  
E. NOTA
- 16) From a certain point on a level plain at the foot of a mountain, the angle of elevation of the peak is  $45^\circ$ . From a point 50 feet farther away, the angle of elevation of the peak is  $30^\circ$ . What is the height (in feet) of the peak above the foot of the mountain?
- A.  $\frac{50}{\sqrt{3}}$   
B.  $50(\sqrt{3} - 1)$   
C.  $\frac{50}{\sqrt{3} - 1}$   
D.  $50\sqrt{3} + 1$   
E. NOTA
- 17) If  $f(x) = \frac{5}{6x+7}$  and  $g(x) = \frac{5-7x}{6x} \left( x \neq 0, -\frac{7}{6} \right)$ , then what is  $f(f(g(f(g(f(f(2)))))))$ ?
- A. 2  
B.  $\frac{4}{3}$   
C.  $\frac{5}{4}$   
D. Does not exist  
E. NOTA
- 18) Billy can paint a house in 12 days. Joe can paint one in 6 days. Jimmy can paint one in 8 days. Billy works everyday, while Joe and Jimmy alternate days. If Joe works on the first day, who is working when the house is completed painted and on which day?
- A. Jimmy, day 4  
B. Jimmy, day 5  
C. Joe, day 4  
D. Joe, day 5  
E. NOTA

- 19) Add:  $2007^2 + 76267 + 361$
- A. 2,529,241  
 B. 4,104,677  
 C. 4,523,562  
 D. 8,563,251  
 E. NOTA
- 20) What point results from the rotation of  $(-\sqrt{27}, 3)$   $60^\circ$  counterclockwise about the origin?
- A.  $(-6, 0)$   
 B.  $(-3\sqrt{3}, -3)$   
 C.  $(-3, -3\sqrt{3})$   
 D.  $(-3\sqrt{3}, -3\sqrt{3})$   
 E. NOTA
- 21)  $t = -9$  is a root of the equation  $t^2 + 4t - 45$ . Which of the following statements is(are) correct for the equation?
- I.  $t - 9$  is a factor of the equation  
 II. Division of the equation by  $t - 9$  yields the other factor of the quadratic equation  
 III.  $t = -5$  is another root of the equation
- A. I only.  
 B. II and III only.  
 C. III only.  
 D. I, II, and III.  
 E. NOTA
- 22) Harold is very athletic. At lunch, he and 4 friends always sit at the same *circular* table with 6 seats (one seat is always empty). Harold will play Ultimate Frisbee with the person/people sitting to the immediate right and left of him. What is the probability that Chris, one of the four friends, will play with him today?
- A. 0.2  
 B. 0.25  
 C. 0.4  
 D. 0.5  
 E. NOTA
- 23) The fourth term of  $(1 - 2x^2)^9$  is:
- A.  $-223x^5$   
 B.  $-1249x^6$   
 C.  $4x^6$   
 D.  $-672x^6$   
 E. NOTA
- 24)  $f(x) = Ax^2 + Bx + C$ . If the sum of the roots is 17, and the product of the roots is 8, then what is the value of  $\frac{B}{C}$ ?
- A.  $-\frac{17}{8}$   
 B.  $\frac{8}{17}$   
 C.  $-\frac{8}{17}$   
 D.  $-\frac{17}{13}$   
 E. NOTA

25)



Ashley is 3 feet tall, and is standing near a 12 foot lamp pole, creating a shadow. Initially, her head is 15 feet from the light source. However, Ashley becomes angry because her shadow is too short, so she walks over and kicks the pole over  $30^\circ$  in the opposite direction. Ashley then walks back to her original position. How long is Ashley's new shadow (in feet)?

A. 4

B.  $\frac{18(2\sqrt{3}-1)}{11}$

C.  $\frac{18(2\sqrt{3}+1)}{11}$

D.  $\frac{18(2\sqrt{3}+1)}{13}$

E. NOTA

**Tiebreakers:**

TB1: Simplify:  $\frac{\sin \frac{\pi}{3} + \cos \frac{2\pi}{3}}{\tan \frac{7\pi}{4}}$

TB2: If  $p$  is a prime number, must  $2^p - 1$  also be prime (yes or no)?

TB3: Multiply:  $\frac{-10}{-9} \cdot \frac{-8}{-7} \cdot \frac{-6}{-5} \cdot \dots \cdot \frac{6}{7} \cdot \frac{8}{9} \cdot \frac{10}{11}$

## ANSWER KEY – ALGEBRA I

1. D
2. B
3. C
4. C
5. B
6. D
7. B
8. C
9. C
10. D
11. C
12. A
13. B
14. D
15. A
16. C
17. A
18. D
19. B
20. B
21. E
22. C
23. D
24. A
25. C

$$\text{TB1 } \frac{1-\sqrt{3}}{2}$$

TB2 No

TB3 0