

20
05

ALABAMA

STATEWIDE MATHEMATICS CONTEST



First Round: March 19, 2005

Second Round: April 23, 2005 at The University of Alabama

ALGEBRA II WITH TRIGONOMETRY EXAM

Construction of this test directed

by

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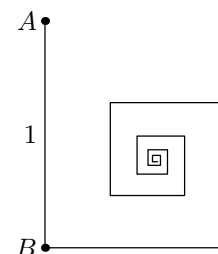
1. With how many zeros does $10^{20}20^{10}$ end?
 (A) 10 (B) 20 (C) **30** (D) 200 (E) 10^{10}

2. $\left(\sqrt[7]{\sqrt[10]{\sqrt[7]{7}}}\right)^{35} =$
 (A) $\frac{1}{7}$ (B) **$\sqrt{7}$** (C) 7 (D) $7\sqrt{7}$ (E) $\frac{\sqrt{7}}{7}$

3. How many solutions does the equation $|2x - 2| = x$ have?
 (A) None (B) 1 (C) **2** (D) 3 (E) Infinite many

4. In the figure, the length of side AB is 1, and for each consecutive side, its length is $\frac{4}{5}$ of the length of previous side. What is the total length of all infinitely many sides?

- (A) 3 (B) 4 (C) **5** (D) 8 (E) Not a finite number



5. The cubes of 2 consecutive positive integers differ by 331. What is the product of the two integers?
 (A) 56 (B) **110** (C) 132 (D) 552 (E) 306

6. Two executives are driving separately to a conference. The first leaves at 2 pm, traveling at 60 mph. The second leaves at 2 : 30 pm. What is the speed of travel of the second executive, if he was passing the first one at 4 : 30 pm?

- (A) 65 mph (B) 70 mph (C) **75 mph** (D) 80 mph (E) 90 mph

7. The scale of a map states that 1 cm represents 150 meters. How many square kilometers would be represented by an area on the map of 80 cm^2 ?

- (A) **1.8 km^2** (B) 2.2 km^2 (C) 14.85 km^2 (D) 230 km^2 (E) 1085 km^2

8. Find the distance from the point $(2, 3)$ to the line $x - y = 5$.

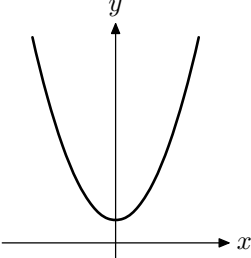
- (A) 1 (B) $\frac{3}{2}$ (C) $\frac{2}{3}$ (D) **$3\sqrt{2}$** (E) $5\sqrt{3}$

9. Suppose that for any integer n ,

$$f(n) = \begin{cases} n - 1, & \text{if } n \text{ is even;} \\ 2n, & \text{if } n \text{ is odd.} \end{cases}$$

If $k \in \mathbb{N}$, and $f(f(f(k))) = 21$, find the sum of the digits in k .

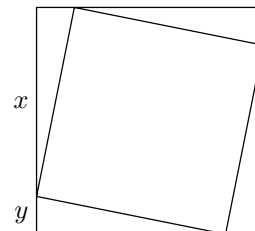
- (A) **3** (B) 4 (C) 5 (D) 7 (E) 11

10. Solve for n if $4 \cdot 2^{n-1} = 8^n$.
- (A) $\frac{1}{4}$ (B) $\boxed{\frac{1}{2}}$ (C) $\frac{5}{2}$ (D) 32 (E) No solution
11. Define $\langle a, b \rangle$ by $\langle a, b \rangle \equiv \frac{a}{a+b}$. If $\langle s, t \rangle = 5$, what is $\langle t, s \rangle$?
- (A) -7 (B) -5 (C) $\boxed{-4}$ (D) 3 (E) 2
12. For how many values of k does the equation $kx^2 + 2kx + 1 = 0$ has exactly one solution?
- (A) None (B) $\boxed{1}$ (C) 2 (D) 3 (E) 4
13. Suppose we know that equation $kx^2 + 4kx + 4 = 0$ has exactly one solution. Then what's the solution?
- (A) $\boxed{-2}$ (B) $-\frac{1}{2}$ (C) $1 + \sqrt{2}$ (D) $\frac{1}{2}$ (E) 1
14. The number $\sqrt{18 + \sqrt{308}}$ can be written in the form $\sqrt{a} + \sqrt{b}$, where a and b are integers and $a > b$. What is the value of $a - b$?
- (A) 3 (B) $\boxed{4}$ (C) 7 (D) 23 (E) 11
15. Which of the following sets of restrictions is true for the function $f(x) = ax^2 + bx + c$ represented by the graph at right?
- (A) $a > 0, b < 0, c > 0$ (B) $a < 0, b > 0, c < 0$
 (C) $a > 0, b < 0, c < 0$ (D) $a < 0, b < 0, c > 0$ (E) $\boxed{\text{None of these}}$
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16. Shifting the graph of $y = \ln x$ vertically is equivalent to which other type of Transformation?
- (A) No other (B) $\boxed{\text{A horizontal stretch}}$ (C) A vertical stretch
 (D) A horizontal shift (E) A rotation about the origin
17. A line has y -intercept $(0, 3)$ and is perpendicular to the line $2x + y = 3$. Find the x -intercept of the line.
- (A) $(6, 0)$ (B) $\boxed{(-6, 0)}$ (C) $(3, 0)$ (D) $(-4, 0)$ (E) $\left(\frac{5}{2}, 0\right)$
18. Find the coefficient of the term containing x^3 in the expansion of $\left(x^2 - \frac{1}{x^3}\right)^9$.
- (A) -72 (B) 72 (C) 84 (D) $\boxed{-84}$ (E) None of these

19. Let $i = \sqrt{-1}$. Which of the following is equivalent to $\frac{3-2i}{4+5i}$?
- (A) $\frac{3}{4} + \frac{2}{5}i$ (B) $\frac{5-4i}{2+3i}$ (C) $\frac{2}{41} - \frac{23}{41}i$ (D) $\frac{7}{81} + \frac{21}{81}i$ (E) $\frac{21}{80} - \frac{71}{80}i$
20. Which of the following is the inverse function of the function $y = \frac{e^x}{5}$?
- (A) $y = \ln x + \ln 5$ (B) $y = 5 \ln x$ (C) $y = \frac{1}{5} \ln x$ (D) $y = \ln(x+5)$ (E) None of these
21. Steve can mow his entire lawn in 40 minutes less time with his power mower than he can with his push mower. One day the power mower broke after he had been mowing for 20 minutes. He finished the lawn with push mower in another 20 minutes. How many minutes does it take Steve to mow his entire lawn with the power mower?
- (A) 30 min (B) 25 min (C) $20\sqrt{2}$ min (D) $10\sqrt{3}$ min (E) 35 min
22. Solve for x if $\frac{1}{\sqrt{x}} + \frac{1}{x + \sqrt{x}} = 1$.
- (A) $\boxed{2}$ (B) 3 (C) 5 (D) 7 (E) $\frac{1}{2}$
23. Which of the following is **NOT** a factor of $x^6 - 1$?
- (A) $x^2 + 1$ (B) $x - 1$ (C) $x^3 + 1$ (D) $x^2 + x + 1$ (E) $x^2 - 1$
24. Find the domain of the function $f(x) = \sqrt{\log(x-1)}$.
- (A) R (B) \emptyset (C) $[0, \infty)$ (D) $[1, \infty)$ (E) $\boxed{[2, \infty)}$
25. The sides of a right triangle are α , $\alpha + \beta$ and $\alpha + 2\beta$ where α and β are real positive numbers. Find the value of $\frac{\alpha}{\beta}$.
- (A) 1 (B) $\frac{3}{2}$ (C) $\frac{8}{3}$ (D) 2 (E) $\boxed{3}$
26. Given $f\left(\frac{x}{3x+5}\right) = 2x$. Find $f(1)$.
- (A) -7 (B) $\boxed{-5}$ (C) -1 (D) 2 (E) 6
27. The points $(4, 7)$ and $(-2, -1)$ lie on opposite ends of a diameter of a circle. What is the distance between the points where the circle intersects the x -axis?
- (A) 5 (B) 6 (C) 7 (D) $\boxed{8}$ (E) 9
28. Solve $3^{(27^x)} = 27^{(3^x)}$ for x .
- (A) $\frac{1}{3}$ (B) $\boxed{\frac{1}{2}}$ (C) 2 (D) 3 (E) 6

29. If $i = \sqrt{-1}$, what is i^{2005} ?
- (A) i (B) $-i$ (C) 1 (D) -1 (E) 0

30. A square is inscribed in another square, such that each vertex divides a side of the outside square into intervals of length x and y , where $x > y$. What is $\frac{x}{y}$, if the area of the inscribed square is $\frac{4}{5}$ of the area of the outside square?



- (A) $4 - \sqrt{15}$ (B) $2 + \sqrt{6}$ (C) $\frac{\sqrt{5}}{2}$ (D) $4 + \sqrt{15}$ (E) $2 - \sqrt{6}$
31. The sum of 30 consecutive integers is 315. What is the largest integer in the list?
- (A) 25 (B) 31 (C) 34 (D) 39 (E) 41

32. How many different 'words' can be made by rearranging the letters in the word POTPOURRI? (The 'words' do not have to make sense)

- (A) 14448 (B) 45360 (C) 4984 (D) 1620 (E) 1290856
33. Find the sum of the following infinite series: $\frac{1}{7} - \frac{5}{7^2} + \frac{1}{7^3} - \frac{5}{7^4} + \frac{1}{7^5} - \frac{5}{7^6} + \dots$
- (A) $\frac{1}{7}$ (B) $\frac{1}{24}$ (C) $\frac{2}{49}$ (D) 0 (E) $\frac{1}{5}$

34. Suppose that Q is an angle that terminates in the fourth quadrant such that $\tan Q = \frac{-1}{3}$. Find $\sin Q$.

- (A) $-\frac{1}{\sqrt{10}}$ (B) $-\frac{1}{\sqrt{3}}$ (C) $\frac{\sqrt{2}}{3}$ (D) $\frac{1}{9}$ (E) $\frac{1}{\sqrt{3}}$
35. Which of the following is equivalent to $3^{\log_4 5}$?
- (A) $3^{\log_5 4}$ (B) $5^{\log_3 4}$ (C) $5^{\log_4 3}$ (D) $4^{\log_3 5}$ (E) $4^{\log_5 3}$

36. If you have three cents, two nickels and one dime, how many different amounts of money can you make using one or more of these coins?

- (A) 19 (B) 20 (C) 6 (D) 33 (E) 28
37. If the product of 2 positive numbers is 9, then possible value of the sum of their reciprocals is in:
- (A) $\left[\frac{1}{3}, \infty\right)$ (B) $[1, \infty)$ (C) $(0, \infty)$ (D) $\left[\frac{2}{3}, \infty\right)$ (E) $\left[\frac{4}{9}, \infty\right)$

38. Find the sum of the solutions of the equation $\log_a (\log_5 x^2) = 2$, where $a = \log_5 x$.

- (A) 41 (B) 26 (C) 16 (D) 19 (E) 7

49. A city council is composed of 6 men and 3 women. Four members are to be chosen as delegates. In how many ways can exactly 2 men and 2 women be chosen?
- (A) 35 (B) 40 (C) 45 (D) 60 (E) 90
50. How many ordered pairs of natural numbers (n, m) are there that solve the equation $2n + 3m = 100$.
- (A) 13 (B) 15 (C) 16 (D) 18 (E) 21