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05

ALABAMA

STATEWIDE MATHEMATICS CONTEST



First Round: March 19, 2005

Second Round: April 23, 2005 at The University of Alabama

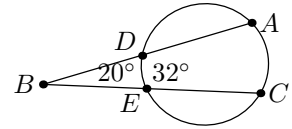
GEOMETRY EXAMINATION

**Construction of this test directed
by**

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1. Given that $m\angle B = 20^\circ$ and $m\widehat{DE} = 32^\circ$. What is the measure of minor arc AC , in degrees?

(A) 52° (B) 64° (C) 72° (D) 84° (E) 110°

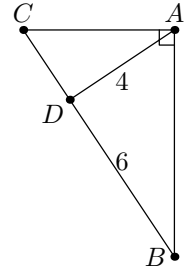


2. What is the circumference, in meters(M), of a circle with area 2004π square meters?

(A) $1002\pi M$ (B) $2\sqrt{501}\pi M$ (C) $501\pi M$ (D) $4\sqrt{501}\pi M$ (E) $2004\pi M$

3. What is length of \overline{BC} in the right triangle $\triangle ABC$ if $\overline{AD} \perp \overline{BC}$?

(A) $\frac{26}{3}$ (B) 52 (C) $\frac{10\sqrt{13}}{3}$ (D) $\frac{8\sqrt{13}}{3}$ (E) $\frac{13}{3}$

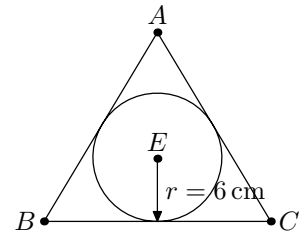


4. A regular polygon has interior angles of 168° . How many sides does the polygon have?

(A) 10 (B) 15 (C) 20 (D) 25 (E) 30

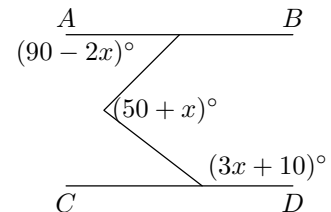
5. A circle is inscribed in an equilateral triangle, the radius of the circle is 6 cm. Find the area of the triangle.

(A) $18\sqrt{3}$ (B) $36\sqrt{3}$ (C) $72\sqrt{3}$ (D) $108\sqrt{3}$ (E) $216\sqrt{3}$



6. Lines \overleftrightarrow{AB} and \overleftrightarrow{CD} are parallel. What is the value of x in degrees?

(A) 25° (B) 35° (C) 45° (D) 55° (E) 65°



7. Let $\triangle ABC$ be an equilateral triangle with side length of 6. Let P be the point of intersection of the three angle bisectors. Find the length of \overline{AP} ?

(A) $2\sqrt{3}$ (B) $\sqrt{3}$ (C) $3\sqrt{3}$ (D) $5\sqrt{3}$ (E) $4\sqrt{3}$

8. A cube of black material is painted blue on all sides, then cut into 216 smaller cubes. How many of the smaller cubes are blue on only one side?

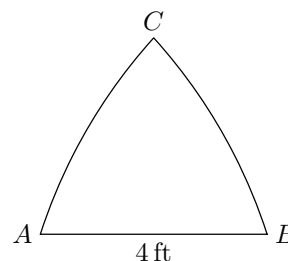
(A) 8 (B) 12 (C) 48 (D) 96 (E) 108

9. How many 8 cm by 8 cm tiles would it take to completely tile the floor of a 44 m by 52 m room?

(A) 115,000 (B) 175,000 (C) 235,000 (D) 300,000 (E) 357,500

18. Determine the area under an archway that was constructed on a straight base AB whose length is 4 feet and with circular arcs AC and BC .

- (A) $\frac{16\pi}{3} \text{ ft}^2$ (B) $\left(\frac{16\pi}{3} - 4\sqrt{3}\right) \text{ ft}^2$ (C) $\frac{8\pi}{3} \text{ ft}^2$
 (D) $\left(\frac{16\pi}{3} + 4\sqrt{3}\right) \text{ ft}^2$ (E) $\left(\frac{8\pi}{3} - 2\sqrt{3}\right) \text{ ft}^2$



19. Find the area of the triangle formed by the coordinate axes and the line $15x - 6y = 90$.

- (A) 30 (B) 36 (C) 40 (D) $\boxed{45}$ (E) 60

20. The perimeter of an isosceles right triangle is $2k$. What is its area?

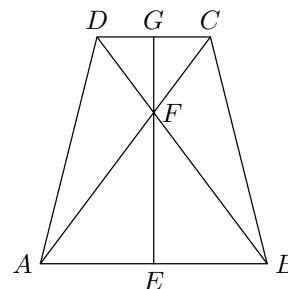
- (A) $4k^2(1 + \sqrt{3})$ (B) $4\sqrt{2}k^2$ (C) $\frac{4k^2 + 3}{3}$ (D) $\frac{3k^2}{2}$ (E) $\boxed{k^2(3 - 2\sqrt{2})}$

21. What is the total surface area of a cylindrical can whose radius is 6 in and whose height is 10 in?

- (A) $156\pi \text{ in}^2$ (B) $\boxed{192\pi \text{ in}^2}$ (C) $136\pi \text{ in}^2$ (D) $36\pi + 60 \text{ in}^2$ (E) $188\pi \text{ in}^2$

22. In the figure shown, the isosceles trapezoid $ABCD$ has bases $AB = 10$ and $CD = 6$. If the diagonals \overline{AC} and \overline{DB} intersect in point F and the altitude \overline{GE} , of length 8 passes through F , then what is the perimeter of the trapezoid $ABCD$?

- (A) 32 (B) $8\sqrt{17}$ (C) $\boxed{16 + 4\sqrt{17}}$ (D) 45 (E) $24 + 8\sqrt{17}$



23. Let the area of a triangle T be numerically equal to its perimeter. Determine the radius of the inscribed circle.

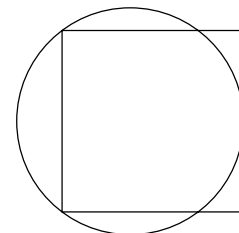
- (A) $\boxed{2}$ (B) 3 (C) 4 (D) 5 (E) 6

24. The smallest angle of a rhombus is half the measure of the larger angle. The shorter diagonal is 20 cm. Determine the perimeter of the rhombus.

- (A) 40 cm (B) 60 cm (C) $\boxed{80 \text{ cm}}$ (D) 100 cm (E) 120 cm

25. In the figure shown, a circle passes through two adjacent vertices of a square and is tangent to the opposite side of the square. If the side length of the square is 3, what is the area of the circle?

- (A) $\frac{9}{4}\pi$ (B) $\frac{16}{9}\pi$ (C) 6π (D) $\frac{36}{25}\pi$ (E) $\boxed{\frac{225}{64}\pi}$

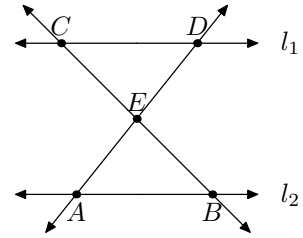


26. A circle goes through the points $(8, 0)$, $(0, 6)$ and $(0, 10)$. Find the x -coordinate of the center of the circle.

- (A) 6.35 (B) 7.15 (C) 7.45 (D) 7.55 (E) $\boxed{7.75}$

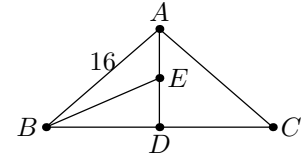
27. In the given figure, if $l_1 \parallel l_2$ and $\overline{AB} \cong \overline{CD}$. Then

- (A) $\overline{AD} \perp \overline{BC}$ (B) $\overline{AC} \cong \overline{BD}$
 (C) $\overline{AE} \cong \overline{EC}$ (D) $\overline{BC} \cong \overline{DE}$ (E) $\overline{AD} \cong \overline{BC}$



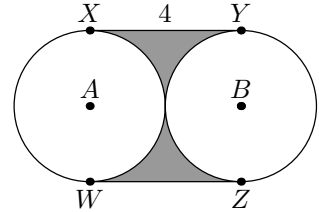
28. In equilateral triangle ABC with side 16. If $\overline{AD} \perp \overline{BC}$ and $\overline{AE} \cong \overline{ED}$, then $BE = ?$

- (A) $4\sqrt{7}$ (B) $2\sqrt{7}$ (C) 96 (D) $\frac{16}{3}\sqrt{3}$ (E) $8\sqrt{6}$



29. In diagram shown, \overline{XY} and \overline{WZ} are common external tangents to equal circles A and B . If $\overline{XY} = 4$, then the area of the shaded region is

- (A) $16 - \frac{\pi}{4}$ (B) $16 + 4\pi$ (C) $8 - 4\pi$ (D) $16 + \frac{\pi}{4}$ (E) $16 - 4\pi$

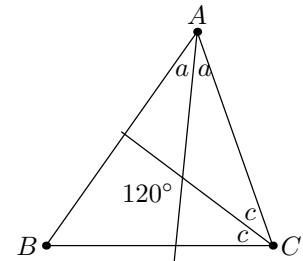


30. Consecutive angles of a pentagon form an arithmetic sequence, if the smallest angle is 100° , then the largest angle is

- (A) 104° (B) 108° (C) 116° (D) 120° (E) 124°

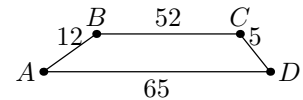
31. In the figure shown, the measure of an angle formed by the bisectors of two angles in triangle ABC is 120° . Find the measure of angle B .

- (A) 40° (B) 45° (C) 50° (D) 60° (E) 80°



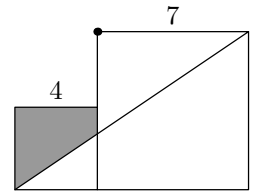
32. In the trapezoid shown, $AD = 65$, $BC = 52$, $AB = 12$, and $CD = 5$. Find the area of the trapezoid.

- (A) 190 (B) 210 (C) 240 (D) 270 (E) 310



33. In the figure shown, the two squares have side length of 4 and 7 respectively. Determine the area of the shaded region.

- (A) $\frac{120}{11}$ (B) $\frac{141}{11}$ (C) $\frac{180}{11}$ (D) $\frac{210}{11}$ (E) $\frac{360}{11}$

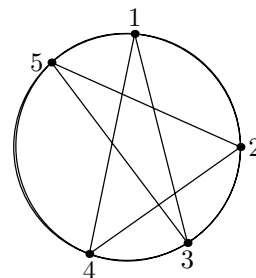


34. In right triangle ABC , the point D on \overline{AB} is 8 units from A , $\angle BAC = 30^\circ$ and $\angle BDC = 60^\circ$. What is the area of the right triangle ABC ?

- (A) 4 (B) $6\sqrt{3}$ (C) 12 (D) $\sqrt{20}$ (E) $24\sqrt{3}$

35. Five points are connected on a circle in the figure shown. What is the sum $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5$?

(A) 90° (B) 180° (C) 270° (D) 360° (E) 540°



36. Let A be the ratio of the volume of a sphere to the volume of a cube each of whose face is tangent to the sphere, and let B be the ratio of the surface area of this sphere to the surface area of the cube. Then

(A) $A + B = \frac{\pi}{6}$ (B) $A + B = \pi$ (C) $A + B = \frac{2\pi}{3}$ (D) $A + B = \frac{\pi}{3}$ (E) $A + B = 2\pi$

37. The sides of a triangle are $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{11}$. Which of the following best describes the triangle?

(A) Isoscales (B) Non-existent (C) Acute (D) Equilateral (E) Scalere

38. Two telephone poles, a and b feet in length are placed x feet apart. Lines are drawn from the top of each pole to the bottom of each other. How many feet above the ground will the two lines intersect?

(A) $\frac{2ab}{a+b}$ (B) $\frac{ab}{2a+2b}$ (C) $\frac{ab}{a+b}$ (D) $\frac{2ab}{a-b}$ (E) $\frac{ab}{a-b}$

39. How many different chords are determined by 8 distinct points lying on a circle?

(A) 40 (B) 18 (C) 32 (D) 28 (E) 48

40. The larger of two similar pyramids has 8 times the volume of the smaller. If the smaller pyramid is 5 inches high, how high is the larger pyramid?

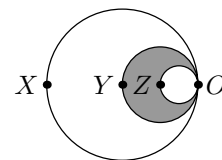
(A) 5 inches (B) 10 inches (C) 20 inches (D) 40 inches (E) 80 inches

41. A sphere is inscribed in a cone whose radius is 6 units and whose height is 8 units. What is volume of the sphere?

(A) $36\pi U^3$ (B) $\frac{32\pi}{3} U^3$ (C) $27\pi U^3$ (D) $\frac{4\sqrt{3}}{3}\pi U^3$ (E) $9\pi U^3$

42. In the figure shown, three circles X , Y and Z are tangent to each other at point O . The center of Y is on Z and the center of X is on Y . If the radius of Z is r , what is the area of the unshaded region?

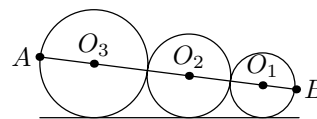
(A) $2\pi r^2$ (B) $3\pi r^2$ (C) $4\pi r^2$ (D) $13\pi r^2$ (E) $16\pi r^2$



43. The volume of a cube with edge $2\sqrt{3}$ is how many times the volume of a cube with edge $\sqrt[3]{3}$?

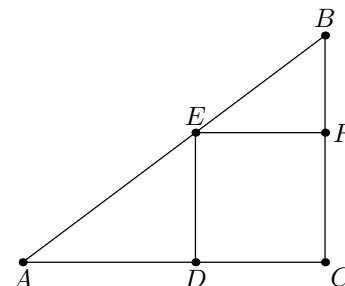
(A) $3\sqrt{3}$ (B) 3 (C) $6\sqrt{3}$ (D) $\sqrt[3]{3}$ (E) $8\sqrt{3}$

44. Three circles are arranged in a row so that each is tangent to the circles next to it. The radii of the two circles at the two ends are 5 and 3. What is the length of the line segment AB that passes through the center of each circle?



- (A) 24 (B) $16 + 4\sqrt{3}$ (C) 20 (D) $16 + 2\sqrt{15}$ (E) $16 + 3\sqrt{15}$

45. In the right triangle ABC shown, $AC = 4$ and $BC = 3$. $CDEF$ is a square. What is the length of EF ?



- (A) $\frac{12}{7}$ (B) 2 (C) $\frac{15}{7}$ (D) $\frac{11}{6}$ (E) $\frac{9}{5}$

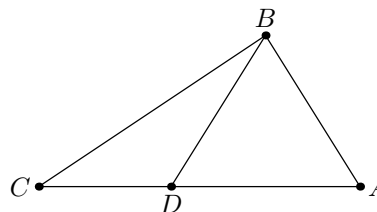
46. In the circle O , chords CD and AB intersect at right angles at point E . If $CE = 8$, $DE = 6$, and $AE = 24$, What is the radius of circle O ?

- (A) $\sqrt{110}$ (B) 8 (C) $\sqrt{170}$ (D) 13 (E) $\sqrt{130}$

47. Suppose a lifeguard is standing at the top of a stand which is built on the edge of the ocean. The lifeguard is looking at a seagull and the lifeguard's eye level is 50 feet above sealevel. If the angle of depression from the horizontal to the lifeguard's line of sight is 60° , how far off shore is the seagull?

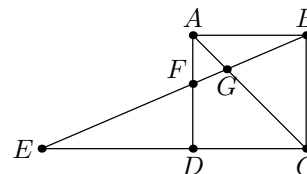
- (A) 25 ft (B) $\frac{50\sqrt{3}}{3}$ ft (C) 50 ft (D) $\frac{100\sqrt{3}}{3}$ ft (E) 100 ft

48. In the figure shown, we are given $AB = 10$, $BC = 14$, $AC = 18$, and D is on \overline{AC} with $BD = 10$. What is the ratio $AD : DC$?



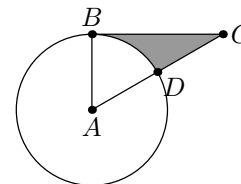
- (A) 14 : 10 (B) 18 : 14 (C) 22 : 12
(D) $38 : 16$ (E) 32 : 10

49. In the square $ABCD$, a line through B intersects the extension of \overline{CD} at E , the side \overline{AD} at F and the diagonal \overline{AC} at G . If $BG = 9$ and $GF = 3$, then what is the length of EF ?



- (A) 12 (B) 24 (C) 18 (D) 36 (E) 48

50. In the figure shown, \overline{BC} is tangent to the circle at point B . If $BC = 6$ and $m\angle ACB = 30^\circ$, then what is the area of the shaded region?



- (A) $6\sqrt{3} - 2\pi$ (B) $\pi - 6$ (C) $12\sqrt{3} - \pi$ (D) $6\sqrt{3} - \pi$ (E) $2\pi - \sqrt{3}$