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STATEWIDE MATHEMATICS CONTEST



First Round : February 18, 2006
 Second Round: April 22, 2006 at The University of Alabama

GEOMETRY EXAMINATION

Construction of this test directed
 by
 Scott H. Brown, Auburn University Montgomery

INSTRUCTIONS

This test consists of 50 multiple choice questions. The questions have not been arranged in order of difficulty. For each question, choose the best of the five answer choices labeled A, B, C, D, and E.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered, and 0 points for each wrong answer. (Thus a “perfect paper” with all questions answered correctly earns a score of 250, a blank paper earns a score of 50, and a paper with all questions answered incorrectly earns a score of 0.)

Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate one or more of the answer choices as wrong, then it is to your advantage to guess among the remaining choices.

- All variables and constants, except those indicated otherwise, represent real numbers.
- Diagrams are not necessarily to scale.

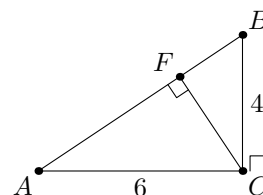
We use the following geometric notation:

- | | |
|--|--|
| • If A and B are points, then: | • If A is an angle, then: |
| \overline{AB} is the segment between A and B | $m \angle A$ is the measure of angle A in degrees |
| \overleftrightarrow{AB} is the line containing A and B | • If A and B are points on a circle, then: |
| \overrightarrow{AB} is the ray from A through B | \widehat{AB} is the arc between A and B |
| AB is the distance between A and B | $m \widehat{AB}$ is the measure of \widehat{AB} in degrees |

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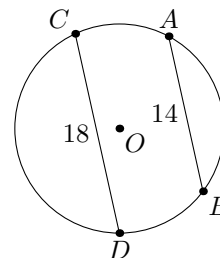
1. In the right triangle $\triangle ABC$, \overline{CF} is perpendicular to \overline{AB} . Find the length of \overline{CF} .

- (A) $\frac{\sqrt{13}}{13}$ (B) $\frac{2\sqrt{13}}{13}$ (C) $\frac{6\sqrt{13}}{13}$ (D) $\frac{12\sqrt{13}}{13}$ (E) $\sqrt{3}$



2. In the figure shown, \overline{AB} is parallel to \overline{CD} and the distance between them is 8. What is the length of the radius of the circle?

- (A) $\sqrt{65}$ (B) $\sqrt{85}$ (C) $\sqrt{75}$ (D) $\sqrt{95}$ (E) 9



3. Given two similar triangles, the area of the larger triangle is sixteen times the area of the smaller triangle. Find the ratio of the perimeter of the larger triangle to the perimeter of the smaller triangle.

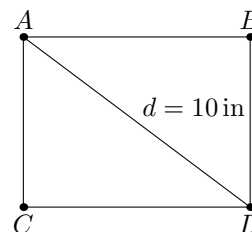
- (A) $\sqrt{8} : 1$ (B) $4 : 1$ (C) $16 : 1$ (D) $32 : 1$ (E) $8 : 1$

4. In a triangle, one angle is three times as large as the other and the third is 20° greater than the sum of the other two. What are the measures of the angles of the triangle?

- (A) $5^\circ, 15^\circ, 160^\circ$ (B) $10^\circ, 30^\circ, 140^\circ$ (C) $20^\circ, 60^\circ, 100^\circ$ (D) $25^\circ, 75^\circ, 80^\circ$ (E) $30^\circ, 60^\circ, 90^\circ$

5. In the figure shown, the area of the rectangle $ABCD$ is 48 in^2 and the length of its diagonal is 10 in. What is the perimeter of the rectangle?

- (A) 8 in (B) 12 in (C) 18 in (D) 20 in (E) 28 in

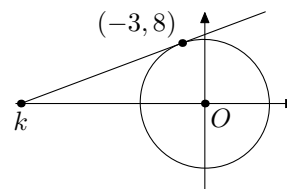


6. Let O_1 and O_2 be concentric circles, with O_2 being the larger circle. The length of a chord of O_2 , which is tangent to O_1 is 36 units. Find the area of the region between O_1 and O_2 .

- (A) $18\pi \text{ unit}^2$ (B) $36\pi \text{ unit}^2$ (C) $81\pi \text{ unit}^2$ (D) $324\pi \text{ unit}^2$ (E) $1296\pi \text{ unit}^2$

7. In the figure shown, a line is tangent to the circle centered at the origin. The point of tangency is $(-3, 8)$. The line intersects the x -axis at $x = k$. Find k .

- (A) $-\frac{82}{3}$ (B) $-\frac{73}{3}$ (C) $-\frac{64}{3}$ (D) $-\frac{16}{3}$ (E) $-\frac{32}{3}$

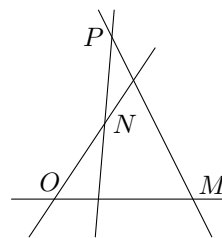


8. What is the area of $\triangle ABC$ with the vertices $A(1, 3)$, $B(1, -5)$ and $C(7, -8)$?

- (A) 12 (B) 20 (C) 24 (D) 36 (E) 48

18. In the figure shown, four straight lines intersect with angles M , N , O , P . What is the value of $M + N + O + P$ in degrees?

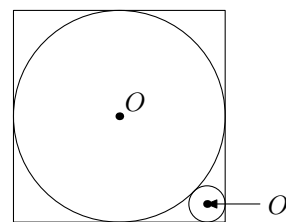
(A) 360° (B) 440° (C) 540° (D) 720° (E) 900°



19. Let circle O be inscribed in a square with side length 1. A smaller circle O' is inscribed in the lower right corner of the square so that O' is tangent to O and the two sides of the square. Find the area of the smaller circle.

(A) $(41 - 24\sqrt{2})\pi$ (B) $\left(\frac{17 + 12\sqrt{2}}{4}\right)\pi$ (C) $(17 + 12\sqrt{2})\pi$

(D) $\left(\frac{17 - 12\sqrt{2}}{4}\right)\pi$ (E) $(41 + 24\sqrt{2})\pi$



20. A dog is tied to the corner of a house with a regular hexagonal base that measures 6 ft on each side. If the rope is 12 ft in length, what is the area in square feet of the region outside the house that the dog can reach?

(A) 108π (B) 144π (C) 180π (D) 216π (E) 256π

21. Tom and Jerry each order a pizza. The circumference of Tom's pizza is 30% greater than the circumference of Jerry's pizza. The area of Tom's pizza is what percentage greater than the area of Jerry's?

(A) 30% (B) 49% (C) 50% (D) 69% (E) 75%

22. Eighteen points $P_1, P_2, P_3, \dots, P_{18}$ are equally spaced on a circle. What is the measure of the angle formed by $P_2P_4P_8$?

(A) 140° (B) 110° (C) 130° (D) 120° (E) 150°

23. If the perimeters of a square and an equilateral triangle are both 3, then by how much more is the area of the square than the area of the triangle?

(A) $\frac{8 - 2\sqrt{3}}{16}$ (B) $\frac{9 - 3\sqrt{3}}{16}$ (C) $\frac{8\sqrt{3} - 11}{16}$ (D) $\frac{6 - 2\sqrt{3}}{16}$ (E) $\frac{9 - 4\sqrt{3}}{16}$

24. Triangle ABC is formed by the vertices $A(8, 3)$, $B(4, 1)$ and $C(-5, 4)$. Determine the length of the altitude to side \overline{AB} .

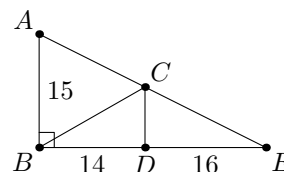
(A) $3\sqrt{5}$ (B) $5\sqrt{5}$ (C) $7\sqrt{5}$ (D) $4\sqrt{5}$ (E) $2\sqrt{5}$

25. Two circles of radius 6 in are externally tangent. What is the length of the tangent from the center of one circle to the other circle?

(A) 6 in (B) $8\sqrt{3}$ in (C) $6\sqrt{3}$ in (D) 12 in (E) 18 in

26. In the right triangle ABE , $\overline{CD} \parallel \overline{AB}$, $AB = 15$, $BD = 14$ and $DE = 16$. What is the length of \overline{BC} ?

(A) $8\sqrt{65}$ (B) 16 (C) $4\sqrt{65}$ (D) 14 (E) $2\sqrt{65}$

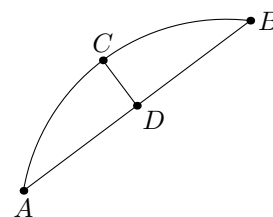


27. Given points $A(8, k)$, $B(5, 2)$ and $C(10, -3)$. Find k so that $\overline{AB} \perp \overline{BC}$.

(A) -5 (B) 3 (C) 1 (D) 5 (E) -3

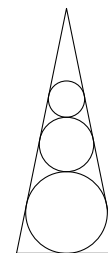
28. In the figure shown, \widehat{AB} is a minor arc of a circle and \overline{CD} is the perpendicular bisector of chord \overline{AB} . If $AB = 40$ and $CD = 8$. Find the circumference of the circle.

(A) 8π (B) 24π (C) 40π (D) 50π (E) 58π



29. In the figure shown, three circles are inscribed in a cone as shown. The radius of the circles are 8, 12, and r . Find the area of largest circle with radius r .

(A) 324π (B) 225π (C) 196π (D) 289π (E) 256π

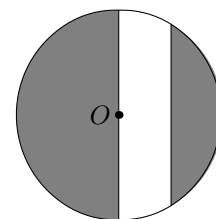


30. A metal strip is shaped into a right triangle with legs of x and $x + 2$ and hypotenuse of $x + 4$. Find the length of the metal strip.

(A) 6 (B) 10 (C) 12 (D) 18 (E) 24

31. A circular sand plot 36 feet in diameter is paved by a straight concrete path 9 feet wide. One edge of the path passes through the center of the plot. Determine the number of square feet in the remaining sand area.

(A) $270\pi - 81\sqrt{3}$ (B) $297\pi - 81\sqrt{3}$
 (C) $275\pi - 81\sqrt{3}$ (D) $250\pi - 81\sqrt{3}$ (E) $154\pi - 81\sqrt{3}$

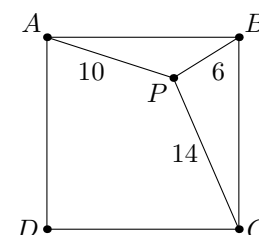


32. The frustum of a cone has a smaller base with a radius of 6 and a larger base with a radius of 10. The length of the lateral segment between the bases is 12. Determine the volume of the cone.

(A) $\frac{500\sqrt{2}\pi}{3}$ (B) $\frac{1000\sqrt{2}\pi}{3}$ (C) $\frac{1500\sqrt{2}\pi}{3}$ (D) $\frac{2000\sqrt{2}\pi}{3}$ (E) $\frac{2500\sqrt{2}\pi}{3}$

33. In the figure shown, point P is located inside square $ABCD$. If $PA = 10$, $PB = 6$ and $PC = 14$, find the area of the square.

(A) $8\sqrt{58}$ (B) 140 (C) 232 (D) $12\sqrt{58}$ (E) 464



34. A cube is inscribed in a sphere. Three vertices of the cube have coordinates $A(3, 4, 1)$, $B(5, 2, 9)$ and $C(1, 6, 5)$. Determine the volume of the sphere.

(A) $72\sqrt{2}\pi$ (B) $144\sqrt{2}\pi$ (C) $288\sqrt{2}\pi$ (D) $576\sqrt{2}\pi$ (E) $1152\sqrt{2}\pi$

35. A circle of diameter 10 is circumscribed by a right-angled isosceles triangle. Determine the perimeter of the triangle.

(A) $20 + 10\sqrt{2}$ (B) $5 + 5\sqrt{2}$ (C) $20 + 20\sqrt{2}$ (D) $30 + 10\sqrt{2}$ (E) $30 + 20\sqrt{2}$

36. Find the volume of a cube with a surface area of 96 cm^2 .

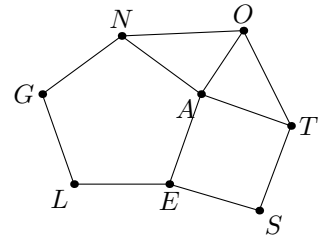
(A) 16 cm^3 (B) 64 cm^3 (C) 96 cm^3 (D) 108 cm^3 (E) 144 cm^3

37. A rectangle has diameter 10. Its length is three times as long as its width. What is its area?

(A) 5 (B) 10 (C) 15 (D) 25 (E) 30

38. In the figure shown, $ANGLE$ is a regular pentagon, $SEAT$ is a square, and OAT is an equilateral triangle. Determine the measure of $\angle TON$.

(A) 39° (B) 99° (C) 117° (D) 139° (E) 151°

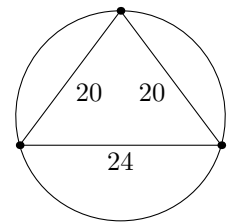


39. A sphere is inscribed in a cube. What is the ratio of the surface area of the sphere to the surface of the cube?

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $\frac{4\pi}{3}$ (D) $\frac{\pi}{12}$ (E) $\frac{8\pi}{3}$

40. In the triangle shown, the lengths of the sides are 20, 20, 24. The triangle is inscribed in a circle. What is the radius of the circle?

(A) 12.5 (B) 13.5 (C) 11.5 (D) 15 (E) 13



41. A square whose area is 64 is partitioned into four congruent smaller squares. Find the circumference of the circle that passes through the centers of the four subsquares.

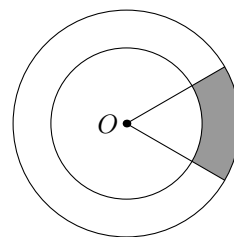
(A) $\frac{\pi}{2}$ (B) 4π (C) $3\sqrt{2}\pi$ (D) $2\sqrt{2}\pi$ (E) $4\sqrt{2}\pi$

42. Container A , of volume a , is one fifth full. Container B , of volume b , is one sixth full. Container C , of volume c , is empty. If all the fluid in the containers is divided equally among the three containers, what part of container C will be full?

(A) $\frac{6a + 5b}{90c}$ (B) $\frac{a + b}{30c}$ (C) $\frac{6a + 5b}{30c}$ (D) $\frac{11ab}{90}$ (E) $\frac{5a + 6b}{90}$

43. The circumferences of two concentric circles are as 2 to 3. Two radii making an angle of 60° intercept a portion of the circular ring which has area of 150 square inches. Determine the radius of the largest circle in terms of π .

- (A) $\frac{\sqrt{5\pi}}{\pi}$ in (B) $\frac{6\sqrt{5\pi}}{\pi}$ in (C) $\frac{900}{\pi}$ in (D) $\frac{18\sqrt{5\pi}}{\pi}$ in (E) $\frac{150}{\pi}$ in



44. Two spherical balls lie on the ground touching. The radius of one ball is 12 ft and the point of contact is 15 ft above the ground. Determine the radius of the other ball.

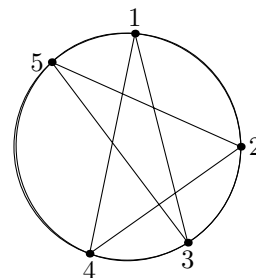
- (A) 12 ft (B) **20 ft** (C) 24 ft (D) 30 ft (E) 36 ft

45. A person walks around an equilateral triangle with side length of 4 ft long. If the person maintains a distance of 4 ft from the triangle at all times, how far must the person walk around fully?

- (A) 12 ft (B) $(8 + 12\pi)$ ft (C) $(6 + 8\pi)$ ft (D) **$(12 + 8\pi)$ ft** (E) 20 ft

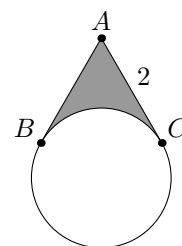
46. Find, in degrees, the sum of angles 1, 2, 3, 4, 5 in the star-shaped figure shown.

- (A) 60° (B) 90° (C) **180°** (D) 270° (E) 360°



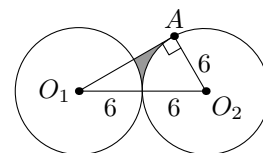
47. In the figure shown, $m\angle A = 60^\circ$, \overline{AB} and \overline{AC} are tangent to the circle. $AC = 2$. Find the area of the shaded region.

- (A) $\frac{3\sqrt{3} - \pi}{2}$ (B) $\frac{4\sqrt{3}}{3} - \pi$ (C) **$\frac{4(3\sqrt{3} - \pi)}{9}$** (D) $\frac{2\sqrt{3} - \pi}{3}$ (E) $3\sqrt{3}$



48. In the figure shown, O_1 and O_2 are centers of the circles. $\overline{O_1A}$ is tangent to the circle centered at O_2 . Find the area of the shaded region.

- (A) $18\sqrt{3} - 7\pi$ (B) $19\sqrt{2} - 8\pi$ (C) **$18\sqrt{3} - 9\pi$**
 (D) $36\sqrt{3} - 15\pi$ (E) None of these



49. Find radius of the circle inscribed in a triangle with sizes 5, 6, and 7, respectively.

- (A) $\frac{4\sqrt{3}}{3}$ (B) **$\frac{2\sqrt{6}}{3}$** (C) $\frac{3\sqrt{3}}{4}$ (D) $\frac{3\sqrt{6}}{4}$ (E) $\frac{4\sqrt{2}}{3}$

50. Given two spheres, the area of the larger sphere is 4 times of the area of the smaller sphere. What is the ratio of the volume of the larger sphere to the volume of the smaller sphere?

- (A) 16 : 1 (B) **8 : 1** (C) 32 : 1 (D) 4 : 1 (E) 2 : 1

What You Can Do With A Mathematics Major

Occupational opportunities

Actuarial and Insurance	Government	Accountant
Computer & Information Sciences	Investment Analyst	Financial Planner
Researcher	Benefits Specialist	Mathematician
Demographers	Computer Programmer	Cartographer
Data Processor	Navigator	Meteorologist
Applications Programmer	Ecologist	Health
Systems Analyst	Biomedical Engineer	Bio-mathematician
Computer Applications Engineer	Operations Analyst	Operations Research
Control Systems Engineer	Control Systems Engineer	Systems Engineer
Numerical Analyst	Teaching	Business Industry
Statistician	Engineering Analyst	Financial Analyst
Technical Writer	Homeland Security	Communications Engineer

Study in the field of mathematics offers an education with an emphasis on careful problem analysis, precision of thought and expression, and the mathematical skills needed for work in many other areas. Many important problems in government, private industry, health and environmental fields, and the academic world require sophisticated mathematical techniques for their solution. The study of mathematics provides specific analytical and quantitative tools, as well as general problem-solving skills, for dealing with these problems. The University of Alabama offers undergraduate and graduate degrees in Mathematics. Please visit www.ua.edu and refer to the undergraduate and graduate programs for additional information.

Engineering Math Advancement Program

The University of Alabama is offering a new summer program to build math skills for students entering engineering. The Engineering Math Advancement Program (EMAP) is a summer residence class that addresses math and engineering prerequisites for incoming engineering students. The program targets bright students who may not have retained the information learned in high school and provides an opportunity to hone technical abilities before entering college. The goal of E-MAP is to assist entering freshmen in developing a solid background in calculus to succeed in engineering before they start at the University.

Classes are designed around Precalculus Algebra and Trigonometry and incorporate important learning principles to ensure that knowledge is retained and not just memorized. Students develop their skills through hands-on experiences, problem solving teaming exercises, and interaction with engineering professors and instructors through an interdisciplinary Living Laboratory program. Experiments allow students to use simple calculus in engineering applications. The program also involves introducing students to local practicing engineers through work on one or more community service engineering-related activities. E-MAP will reserve 33-40 percent of enrollment space for underrepresented groups. Financial assistance is available based on need. Please visit emap.ua.edu for additional information.