2007

ALABAMA

STATEWIDE MATHEMATICS CONTEST

First Round : March 31, 2007
Second Round: April 21, 2007 at The University of Alabama

GEOMETRY EXAMINATION

Construction of this test directed
by
Scott H. Brown, Auburn University Montgomery
and Zhijian Wu, The University of Alabama

INSTRUCTIONS

This test consists of 50 multiple choice questions. The questions have not been arranged in order of difficulty. For each question, choose the best of the five answer choices labeled A, B, C, D, and E.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered, and 0 points for each wrong answer. (Thus a "perfect paper" with all questions answered correctly earns a score of 250, a blank paper earns a score of 50, and a paper with all questions answered incorrectly earns a score of 0.)

Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate one of more of the answer choices as wrong, then it is to your advantage to guess among the remaining choices.

- All variables and constants, except those indicated otherwise, represent real numbers.
- Diagrams are not necessarily to scale.

We use the following geometric notation:

- If A and B are points, then:
  $\overline{AB}$ is the segment between A and B
  $\overleftrightarrow{AB}$ is the line containing A and B
  $\overrightarrow{AB}$ is the ray from A through B
  AB is the distance between A and B
- If A is an angle, then:
  $m\angle A$ is the measure of angle A in degrees
- If A and B are points on a circle, then:
  $\widehat{AB}$ is the arc between A and B
  $m\,\widehat{AB}$ is the measure of $\widehat{AB}$ in degrees

Editing by Zhijian Wu, The University of Alabama
Printing by The University of Alabama

What You Can Do With A Mathematics Major

Occupational opportunities

Actuarial and Insurance	Government	Accountant
Computer & Information Sciences	Investment Analyst	Financial Planner
Researcher	Benefits Specialist	Mathematician
Demographers	Computer Programmer	Cartographer
Data Processor	Navigator	Meteorologist
Applications Programmer	Ecologist	Health
Systems Analyst	Biomedical Engineer	Bio-mathematician
Computer Applications Engineer	Operations Analyst	Operations Research
Control Systems Engineer	Control Systems Engineer	Systems Engineer
Numerical Analyst	Teaching	BusinessIndustry
Statistician	Engineering Analyst	Financial Analyst
Technical Writer	Homeland Security	Communications Engineer

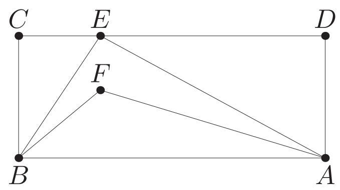
Study in the field of mathematics offers an education with an emphasis on careful problem analysis, precision of thought and expression, and the mathematical skills needed for work in many other areas. Many important problems in government, private industry, health and environmental fields, and the academic world require sophisticated mathematical techniques for their solution. The study of mathematics provides specific analytical and quantitative tools, as well as general problem-solving skills, for dealing with these problems. The University of Alabama offers undergraduate and graduates degrees in Mathematics. Please visit **www.ua.edu** and refer to the undergraduate and graduate programs for additional information.

Engineering Math Advancement Program

The University of Alabama is offering a summer program to build math skills for students entering engineering. The Engineering Math Advancement Program, E-MAP, is a five-week summer residence class that addresses math and engineering prerequisites for incoming engineering students. The program targets bright students who may not have retained the information learned in high school and provides and opportunity to hone technical abilities before entering college. The goal of E-MAP is to assisting entering freshmen in developing a solid background in calculus to succeed in engineering before they start at the University.

Classes are designed around Precalculus Algebra and Trigonometry and incorporate important learning principles to ensure that knowledge is retained and not just memorized. Students develop their skills through hands-on experiences, problem solving teaming exercises, and interaction with engineering professors and instructors through an interdisciplinary Living Laboratory program. Experiments allow students to use simple calculus in engineering applications. The program also involves introducing students to local practicing engineers through work on one or more community service engineering-related activities. E-MAP will reserve 33-40 percent of enrollment space for underrepresented groups. Financial assistance is available based on need. Please visit **www.emap.ua.edu** for additional information.

1. In the figure shown, $m\angle BEA = 100°$. Point F is chosen inside triangle $\triangle BEA$ so that line FA bisects $\angle EAB$ and line FB bisects $\angle EBA$. Find the measure of $\angle BFA$.



(**A**) $\boxed{140°}$ (**B**) 145° (**C**) 150° (**D**) 155° (**E**) 160°

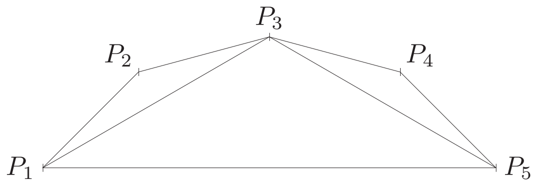
2. Suppose we have two concentric reflective hollow spheres S_1 and S_2 with radius R and $\frac{\sqrt{3}}{13}R$ respectively. From a point P on the surface of S_2, a ray of light is emitted inward at 60° from the radial direction. The ray eventually returns to P. How many total reflections off the two spheres does it take?

(**A**) 11 (**B**) 7 (**C**) $\boxed{5}$ (**D**) 13 (**E**) infinitely many

3. Let P be a point on the circumference of a circle. Perpendiculars PA and PB are drawn to points A and B on two mutually perpendicular diameters. If $AB = 36$ in, what is the diameter of the circle?

(**A**) 8 in (**B**) 16 in (**C**) 24 in (**D**) 36 in (**E**) $\boxed{72\text{ in}}$

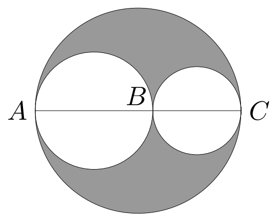
4. In the figure shown, $P_1\,P_2\,P_3\,P_4\,P_5$ is a section of a regular dodecagon with each side length of 2. Find the area of triangle $\triangle P_1P_3P_5$.



(**A**) $\boxed{3+2\sqrt{3}}$ (**B**) $5+4\sqrt{3}$

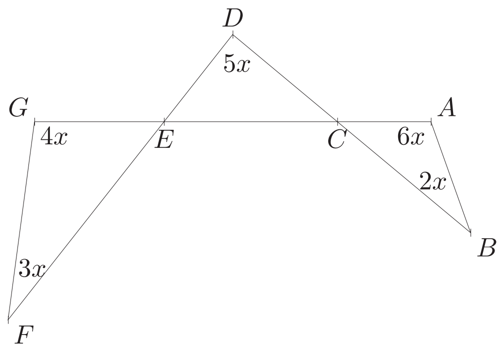
(**C**) $6+2\sqrt{3}$ (**D**) $8+4\sqrt{3}$ (**E**) $8+2\sqrt{3}$

5. In the figure shown, AC has length 7, semicircle $\widehat{AB}$ has radius 2, semicircle $\widehat{BC}$ has diameter 3. What percent of the big circle is shaded (round to the nearest units)?



(**A**) 24% (**B**) 33% (**C**) 40% (**D**) $\boxed{49\%}$ (**E**) 56%

6. In the figure shown, what is the value of $\angle CAB$? (Here x is measured in degree in the figure.)



(**A**) 54° (**B**) $\boxed{108°}$

(**C**) 120° (**D**) 144° (**E**) 162°

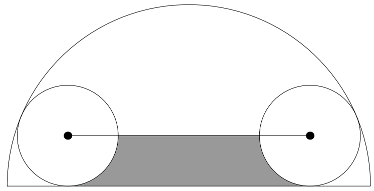
7. The sides of a triangle are in the ratio 4 : 6 : 11. Which of the following words best describes the triangle?

(**A**) right (**B**) obtuse (**C**) isosceles (**D**) acute (**E**) $\boxed{\text{impossible}}$

8. A right triangle with integer side lengths a, b and c satisfies $a < b < c$ and $a + c = 81$. What is the maximum area of the triangle given these conditions?

(**A**) 480 unit^2 (**B**) 504 unit^2 (**C**) 580 unit^2 (**D**) $\boxed{630\text{ unit}^2}$ (**E**) 660 unit^2

9. In the figure shown, two circles of radius 5 are placed inside a semicircle of radius 18. The two circles are tangent to the diameter and to the semicircle. Find the area of the shaded region?



(**A**) $\dfrac{180-35\pi}{2}$ (**B**) $\dfrac{120-25\pi}{2}$ (**C**) $\dfrac{240-35\pi}{2}$

(**D**) $\boxed{\dfrac{240-25\pi}{2}}$ (**E**) $\dfrac{360-35\pi}{2}$

10. The curves relating to the equation $2x^2+2y^2=10$ and $4x^2-2y^2=8$ intersect at four points. The four points are the vertices of a rectangle. What is the area of the rectangle?

(**A**) $6\,\text{unit}^2$ (**B**) $\sqrt{6}\,\text{unit}^2$ (**C**) $\boxed{4\sqrt{6}\,\text{unit}^2}$ (**D**) $8\,\text{unit}^2$ (**E**) $24\,\text{unit}^2$

11. On the parabola $y=4x^2+8x-4$ lies points A and B. The origin of the coordinate system is the midpoint of the line segment joining A and B. What is the length of this line segment?

(**A**) $\boxed{2\sqrt{65}}$ (**B**) $\sqrt{85}$ (**C**) $4\sqrt{65}$ (**D**) $2\sqrt{85}$ (**E**) $\sqrt{65}$

12. In the triangle ABC, we have $\angle ACB=120°$, $AC=8$ and $BC=4$. The angle bisector of $\angle ACB$ meets the side AB at the point D. Find the length of CD.

(**A**) $\dfrac{2}{3}$ (**B**) 1 (**C**) $\dfrac{4}{3}$ (**D**) 2 (**E**) $\boxed{\dfrac{8}{3}}$

13. A line l_1 has a slop of -4 and passes through the point $(r,-2)$. A second line l_2, is perpendicular to l_1, intersects at the point (a,b), and passes through the point $(8,r)$. The value of a is

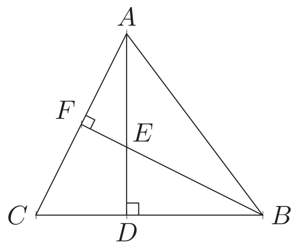
(**A**) $\boxed{\dfrac{12}{17}r}$ (**B**) 1 (**C**) $\dfrac{19}{17}r$ (**D**) $r-2$ (**E**) $\dfrac{21}{17}r$

14. The sum of the length of the three sides of a right triangle is 26. The sum of the squares of the lengths of the three sides is 288. Determine the area of the triangle.

(**A**) $\boxed{13}$ (**B**) 15 (**C**) 18 (**D**) 26 (**E**) 52

15. In the triangle shown, altitude AD meets altitude BF at E. Suppose $AD=8$, $BD=6$, and $CD=4$. What is the length of ED?

(**A**) $\dfrac{1}{2}$ (**B**) $\dfrac{\sqrt{5}}{2}$ (**C**) 2 (**D**) $\dfrac{5}{2}$ (**E**) $\boxed{3}$



16. Points A, B and C lie on a circle and form an equilateral triangle. If $AB=12$ what is the circumference of the circle.

(**A**) $\sqrt{3}\,\pi$ (**B**) $4\sqrt{3}\,\pi$ (**C**) $\boxed{8\sqrt{3}\,\pi}$ (**D**) $10\sqrt{3}\,\pi$ (**E**) $12\sqrt{3}\,\pi$

17. The parabola whose equation is $27y=x^2$ meets the parabola whose equation is $x=y^2$ at two points. Determine the distance between these two points.

(**A**) $\sqrt{10}$ (**B**) $\boxed{3\sqrt{10}}$ (**C**) $5\sqrt{10}$ (**D**) $7\sqrt{10}$ (**E**) $8\sqrt{10}$

18. Two lines y and z form a 60° angle at the point X, and Y_1 is a point on y. From Y_1 draw a line perpendicular to the line y meeting the line z at the point Z_1. From Z_1 draw a line perpendicular to z meeting the line y at Y_2. Continue in this manner giving points Z_2, Y_2, Z_3, Y_3 and so on. These points are the vertices of right triangles $XY_1Z_1, XY_2Z_2, \cdots$. If the area of $\triangle XY_1Z_1 = 1$, determine $\text{area}\triangle XY_1Z_1 + \text{area}\triangle XY_2Z_2 + \cdots + \text{area}\triangle XY_{2007}Z_{2007}$.

(**A**) $\dfrac{16^{2007}-1}{7}$ (**B**) $\dfrac{16^{2006}-1}{15}$ (**C**) $\boxed{\dfrac{16^{2007}-1}{15}}$ (**D**) $\dfrac{16^{2007}}{7}$ (**E**) $\dfrac{16^{2006}-1}{15}$

19. The radius of a right circular cylinder is increased by 40% and the height is decreased by 50%. What is the change in the volume?

(**A**) stays the same (**B**) increase by 2% (**C**) decrease by 4%

(**D**) increase by 4% (**E**) $\boxed{\text{decrease by 2\%}}$

20. In a right triangle the length of all three sides are positive integers. If the area of the triangle is 120, then what is the length of the hypotenuse?

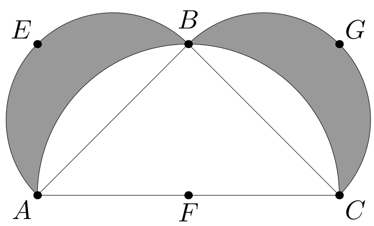
(**A**) 10 (**B**) 13 (**C**) 15 (**D**) 20 (**E**) $\boxed{26}$

21. The surface areas of the six faces of a rectangular solid are 9, 9, 16, 16, 25, 25 square units, respectively. The volume of the solid, in cube units, is:

(**A**) 36 (**B**) 48 (**C**) $\boxed{60}$ (**D**) 72 (**E**) 80

22. In the figure shown, $\widehat{ABC}$, $\widehat{AEB}$, and $\widehat{CGB}$ are semicircles. F is the midpoint of AC. $AF = FC = 1$ unit, and $AB = BC$. What is the area of the shaded region?

(**A**) $\dfrac{1}{2}$ (**B**) $\dfrac{\pi}{8} - \dfrac{1}{2}$ (**C**) $\boxed{1}$ (**D**) $\dfrac{\pi}{4}$ (**E**) $\dfrac{\pi}{4} - \dfrac{1}{2}$

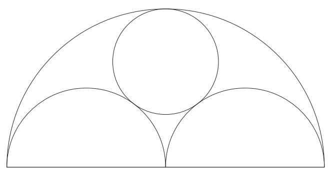


23. The perimeter of rectangle is 64 units. The ratio of its length to width is $5:3$. Determine the length of a diagonal of the rectangle.

(**A**) 12 (**B**) $\boxed{4\sqrt{34}}$ (**C**) 20 (**D**) $8\sqrt{34}$ (**E**) 32

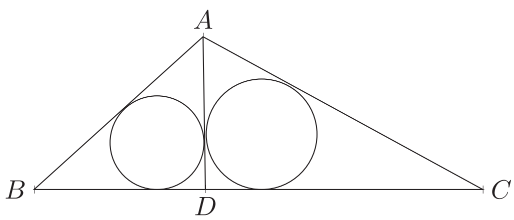
24. Two semicircles of radius 6 units are inscribed in a semicircle of radius 12 units. A circle is inscribed so that it is tangent to all three semicircles. Find the circumference of the circle.

(**A**) $\boxed{8\pi}$ (**B**) 6π (**C**) π (**D**) 4π (**E**) 2π



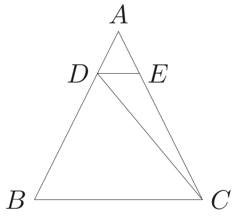
25. In triangle ABC, AB=260, AC=400 and $BC = 520$. Point D is chosen on BC so that the circle inscribed in triangle ABD and ADC are tangent to AD at the same point. What is the length of BD?

(**A**) 130 (**B**) 150 (**C**) 170 (**D**) $\boxed{190}$ (**E**) 210



26. In the triangle ABC, $\overline{DE} \parallel \overline{BC}$ and $DE : BC = 1 : 4$. If the area of $\triangle ADE$ is 20 square units, then the area of $\triangle DEC$ is (in square units):

(**A**) 30 (**B**) 40 (**C**) 50 (**D**) $\boxed{60}$ (**E**) 70

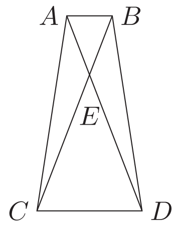


27. What is the perimeter of the triangle whose vertices are $(1,-1)$, $(5,-1)$, $(3,8)$?

(**A**) 20 (**B**) $\boxed{2\sqrt{85}+4}$ (**C**) $2\sqrt{33}-1$ (**D**) $\sqrt{85}+4$ (**E**) $2\sqrt{21}+4$

28. In the figure shown, $\overline{AB} \parallel \overline{CD}$, $AB = 3$. If the area of $\triangle ABE$ is 9 and the area of $\triangle CDE$ is 12, then the length of CD is:

(**A**) $\boxed{2\sqrt{3}}$ (**B**) $\sqrt{3}$ (**C**) $\sqrt{5}$ (**D**) $2\sqrt{5}-1$ (**E**) $\sqrt{15}$

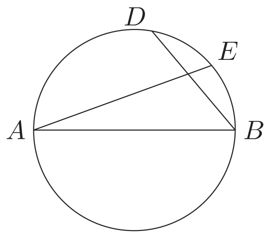


29. Three circles are mutually tangent externally. Their centers form a triangle whose sides are of lengths $3, 4$ and 5. Find the total area of the three circles (in square units).

(**A**) 9π (**B**) 16π (**C**) 25π (**D**) 21π (**E**) $\boxed{14\pi}$

30. In the figure shown, $\overline{AB}$ is a diameter of a circle. If $\angle A = 20°$ and $\angle B = 50°$, what is the measure of the arc $\widehat{DE}$?

(**A**) $60°$ (**B**) $55°$ (**C**) $25°$ (**D**) $\boxed{40°}$ (**E**) $30°$

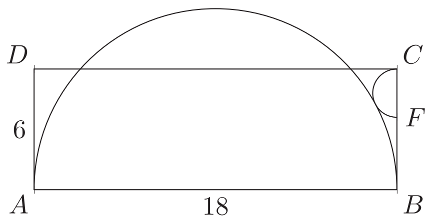


31. The frustum of a cone has a smaller base with a radius of 3 and a larger base with a radius of 5. The length of the lateral segment between the bases is 6. Determine the volume of the cone (in cube units).

(**A**) $\dfrac{100\sqrt{2}\,\pi}{3}$ (**B**) $150\sqrt{2}\,\pi$ (**C**) $\dfrac{200\sqrt{2}\,\pi}{3}$ (**D**) $\boxed{\dfrac{250\sqrt{2}\,\pi}{3}}$ (**E**) $100\sqrt{2}\,\pi$

32. In the figure shown, $ABCD$ is a rectangle, $AB = 18$, $AD = 6$, and $\widehat{AB}$ is a semicircle with diameter AB, $\widehat{CF}$ is a semicircle with diameter CF. These two arcs are tangent to each other. What is the length of FB?

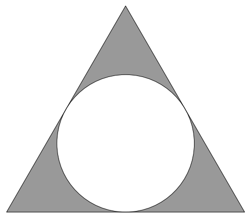
(**A**) $\boxed{\dfrac{18}{5}}$ (**B**) $\dfrac{12}{5}$ (**C**) $\dfrac{21}{5}$ (**D**) 3 (**E**) 4



33. What is the total surface area of a cylindrical can whose radius is 8 in and whose height is 5 in?

(**A**) $156\pi\,\text{in}^2$ (**B**) $\boxed{208\pi\,\text{in}^2}$ (**C**) $136\pi\,\text{in}^2$ (**D**) $185\pi\,\text{in}^2$ (**E**) $144\pi\,\text{in}^2$

34. In the figure shown, a circle is inscribed in an equilateral triangle. The diameter of the circle is 10 units. Find the area of the shaded region (in square units).

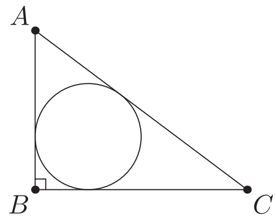


(**A**) $75 + 25\pi$ (**B**) $75 - 25\pi$ (**C**) $\boxed{75\sqrt{3} - 25\pi}$ (**D**) 25π (**E**) $75\sqrt{3}$

35. Find the equation of the perpendicular bisector of the line segment joining points $(7, -1)$ and $(-1, 3)$.

(**A**) $y - 2x = 0$ (**B**) $\boxed{2x - y - 5 = 0}$ (**C**) $x - 2y - 5 = 0$ (**D**) $2x - y = 0$ (**E**) $2x - y + 5 = 0$

36. In the figure shown, a circle is inscribed in $\triangle ABC$. The lengths of three sides of the triangle are 3, 4 and 5. Find the radius of the circle.



(**A**) 2 (**B**) $\boxed{1}$ (**C**) $\frac{3}{2}$ (**D**) $\frac{1}{2}$ (**E**) $\frac{5}{2}$

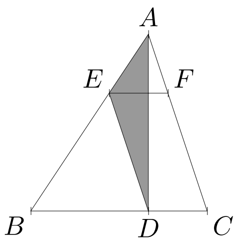
37. Two circles can divide a plane into 4 regions at most. What is the maximum number of regions obtained by dividing a plane with 4 circles?

(**A**) 8 (**B**) 10 (**C**) 12 (**D**) $\boxed{14}$ (**E**) 16

38. Find the equation of the parabola which passes though the three points $(0, 1)$, $(1, 4)$, $(2, 9)$.

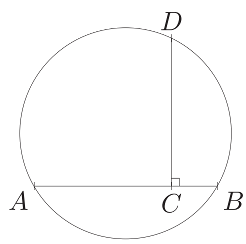
(**A**) $y = x^2$ (**B**) $\boxed{y = x^2 + 2x + 1}$ (**C**) $x = y^2 - 1$ (**D**) $y = x^2 + 3x + 1$ (**E**) $x = y^2 + 2y + 1$

39. In the figure shown, $DEFC$ is a parallelogram with area of $28\,\text{cm}^2$. Points D, E and F lie on $\overline{BC}$, $\overline{AB}$ and $\overline{AC}$, respectively. Find the area of the shaded region (in cm^2).



(**A**) $\boxed{14}$ (**B**) 15 (**C**) 16 (**D**) 17 (**E**) 18

40. In the figure shown, $\overline{AB} \perp \overline{CD}$. Which of the following is true?



I. $m\widehat{AD} = m\widehat{AB}$ II. $m\widehat{BD} = m\widehat{AB}$ III. $AC = CD$

(**A**) I (**B**) II (**C**) III (**D**) I, II, III (**E**) $\boxed{\text{none of these}}$

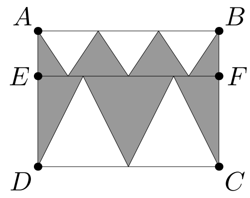
41. Given two spheres, the volume of the larger sphere is 27 times of the volume of the smaller one. What is the ratio of the surface area of the smaller sphere to the surface area of the larger sphere?

(**A**) $6:1$ (**B**) $9:1$ (**C**) $1:3$ (**D**) $1:27$ (**E**) $\boxed{1:9}$

42. Find the number of diagonals in a convex decagon(a polygon with ten sides).

(**A**) 20 (**B**) 30 (**C**) $\boxed{35}$ (**D**) 40 (**E**) 45

43. In the figure shown, $ABCD$ and $CDEF$ are rectangles, $AB = 4$ and $BC = 3$. Find the area of the shaded region (in unit square).

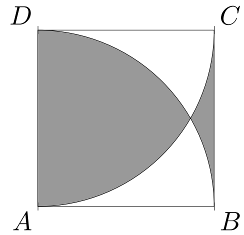


(**A**) 4 (**B**) 5 (**C**) $\boxed{6}$ (**D**) 7 (**E**) 8

44. What is the distance from the point $(2, 1)$ to its symmetric point respect to the line $3x - 4y + 4 = 0$?

(**A**) $\boxed{\frac{6}{5}}$ (**B**) $\frac{8}{5}$ (**C**) 2 (**D**) $\frac{12}{5}$ (**E**) $\frac{14}{5}$

45. In the figure shown, $ABCD$ is a square. $AB = 1$, $\overset{\frown}{AC}$ and $\overset{\frown}{BD}$ are arcs with radius 1 and centers at D and A, respectively. What is the difference between the areas of the two shaded regions?

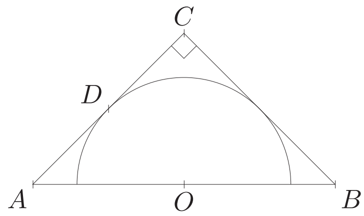


(**A**) $\pi - 1$ (**B**) $1 - \frac{\pi}{4}$ (**C**) $\frac{\pi}{3} - 1$ (**D**) $1 - \frac{\pi}{6}$ (**E**) $\boxed{\frac{\pi}{2} - 1}$

46. A convex polygon is inscribed in a circle with radius 1. If every side of the polygon is bigger than 1 but less than $\sqrt{2}$, how many sides this polygon has?

(**A**) 3 (**B**) 4 (**C**) $\boxed{5}$ (**D**) 6 (**E**) not sure

47. In the figure shown, $\triangle ABC$ is a right triangle. A semicircle with center O is tangent to $\overline{AC}$ and $\overline{BC}$. If the area of $\triangle ABC$ is S and $AB = s$, then the radius of the semicircle is:



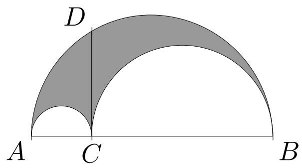
(**A**) $\dfrac{S}{\sqrt{s^2 + 4S}}$ (**B**) $\dfrac{2S}{\sqrt{s^2 + 2S}}$ (**C**) $\dfrac{S}{\sqrt{s^2 + 2S}}$

(**D**) $\boxed{\dfrac{2S}{\sqrt{s^2 + 4S}}}$ (**E**) $\dfrac{2S}{\sqrt{s^2 + S}}$

48. A circle with radius r is inscribed in the triangle ABC which has sides a, b and c. Suppose the area of the $\triangle ABC$ is 5, write r as a function of a, b and c.

(**A**) $\boxed{\dfrac{10}{a+b+c}}$ (**B**) $\dfrac{10}{abc}$ (**C**) $\dfrac{a+b+c}{5}$ (**D**) $\dfrac{5}{abc}$ (**E**) none of these

49. In the figure shown, $\overset{\frown}{AB}$ is a semicircle with diameter $\overline{AB}$. $\overset{\frown}{AC}$ is a semicircle with diameter $\overline{AC}$. $\overset{\frown}{BC}$ is a semicircle with diameter $\overline{BC}$. D is a point on $\overset{\frown}{AB}$ and $\overline{CD} \perp \overline{AB}$. If $CD = 1$, what is the area of the shaded region?



(**A**) $\frac{1}{3}\pi$ (**B**) $\boxed{\frac{1}{4}\pi}$ (**C**) $\frac{1}{5}\pi$ (**D**) $\frac{1}{6}\pi$ (**E**) $\frac{1}{8}\pi$

50. From 2 : 30 pm. to 2 : 50 pm, how many degrees does the hour hand cross?

(**A**) 5° (**B**) $\boxed{10°}$ (**C**) 15° (**D**) 20° (**E**) 25°