2008 ALABAMA

STATEWIDE MATHEMATICS CONTEST

First Round : March 29, 2008
Second Round: April 26, 2008 at The University of Alabama

GEOMETRY EXAMINATION

Construction of this test directed
by
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INSTRUCTIONS

This test consists of 50 multiple choice questions. The questions have not been arranged in order of difficulty. For each question, choose the best of the five answer choices labeled A, B, C, D, and E.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered, and 0 points for each wrong answer. (Thus a "perfect paper" with all questions answered correctly earns a score of 250, a blank paper earns a score of 50, and a paper with all questions answered incorrectly earns a score of 0.)

Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate one of more of the answer choices as wrong, then it is to your advantage to guess among the remaining choices.

- All variables and constants, except those indicated otherwise, represent real numbers.
- Diagrams are not necessarily to scale.

We use the following geometric notation:

- If A and B are points, then:
  $\overline{AB}$ is the segment between A and B
  $\overleftrightarrow{AB}$ is the line containing A and B
  $\overrightarrow{AB}$ is the ray from A through B
  AB is the distance between A and B
- If A is an angle, then:
  $m\angle A$ is the measure of angle A in degrees
- If A and B are points on a circle, then:
  $\overset{\frown}{AB}$ is the arc between A and B
  $m\overset{\frown}{AB}$ is the measure of $\overset{\frown}{AB}$ in degrees

Editing by Zhijian Wu, The University of Alabama
Printing by The University of Alabama

What You Can Do With A Mathematics Major

Occupational opportunities

Actuarial and Insurance	Government	Accountant
Computer & Information Sciences	Investment Analyst	Financial Planner
Researcher	Benefits Specialist	Mathematician
Demographers	Computer Programmer	Cartographer
Data Processor	Navigator	Meteorologist
Applications Programmer	Ecologist	Health
Systems Analyst	Biomedical Engineer	Bio-mathematician
Computer Applications Engineer	Operations Analyst	Operations Research
Control Systems Engineer	Control Systems Engineer	Systems Engineer
Numerical Analyst	Teaching	BusinessIndustry
Statistician	Engineering Analyst	Financial Analyst
Technical Writer	Homeland Security	Communications Engineer

Study in the field of mathematics offers an education with an emphasis on careful problem analysis, precision of thought and expression, and the mathematical skills needed for work in many other areas. Many important problems in government, private industry, health and environmental fields, and the academic world require sophisticated mathematical techniques for their solution. The study of mathematics provides specific analytical and quantitative tools, as well as general problem-solving skills, for dealing with these problems. The University of Alabama offers undergraduate and graduates degrees in Mathematics. Please visit **www.ua.edu** and refer to the undergraduate and graduate programs for additional information.

Engineering Math Advancement Program

The University of Alabama is offering a new summer program to build math skills for students entering engineering. The Engineering Math Advancement Program (EMAP) is a summer residence class that addresses math and engineering prerequisites for incoming engineering students. The program targets bright students who may not have retained the information learned in high school and provides and opportunity to hone technical abilities before entering college. The goal of E-MAP is to assisting entering freshmen in developing a solid background in calculus to succeed in engineering before they start at the University.

Classes are designed around Precalculus Algebra and Trigonometry and incorporate important learning principles to ensure that knowledge is retained and not just memorized. Students develop their skills through hands-on experiences, problem solving teaming exercises, and interaction with engineering professors and instructors through an interdisciplinary Living Laboratory program. Experiments allow students to use simple calculus in engineering applications. The program also involves introducing students to local practicing engineers through work on one or more community service engineering-related activities. E-MAP will reserve 33-40 percent of enrollment space for underrepresented groups. Financial assistance is available based on need. Please visit **emap.ua.edu** for additional information.

1. In a right square pyramid all five faces have the same area. If the height of the pyramid is $9\,in$, what is the total surface area in square inchs?

(**A**) 36 (**B**) 64 (**C**) 81 (**D**) $\boxed{108}$ (**E**) 144

2. Two regular polygons of the same number of sides have sides $48\,m$ and $55\,m$ in length respectively. What is the length of the side of the another regular polygon of the same number of sides, if its area is equal to the sum of the two?

(**A**) $68\,m$ (**B**) $\boxed{73\,m}$ (**C**) $86\,m$ (**D**) $103\,m$ (**E**) $110\,m$

3. A $160\,ft$ rope is suspended at its two ends from the top of the $100\,ft$ poles. If the lowest point to which the midpoint at the rope can be polled from the ground is $72\,ft$, what is the distance between the poles?

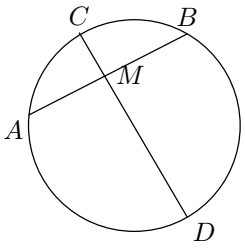
(**A**) $12\sqrt{39}\,ft$ (**B**) $16\sqrt{39}\,ft$ (**C**) $\boxed{24\sqrt{39}\,ft}$ (**D**) $28\sqrt{39}\,ft$ (**E**) $32\sqrt{39}\,ft$

4. A circle with center O has a chord AB equal in length to the radius of 1. A perpendicular from O to AB meets AB at M. A perpendicular from M to OA at D. What is the area of triangle AMD (in unit square)?

(**A**) $\frac{\sqrt{3}}{2}$ (**B**) $\frac{\sqrt{3}}{4}$ (**C**) $\frac{\sqrt{3}}{8}$ (**D**) $\frac{\sqrt{3}}{16}$ (**E**) $\boxed{\frac{\sqrt{3}}{32}}$

5. In the circle shown, $AB = 24$, and the perpendicular chord $\overline{CD}$ bisects $\overline{AB}$. How long is $\overline{BD}$ if $\overline{DM}$ is 4 times as long as $\overline{CM}$?

(**A**) $4\sqrt{5}$ (**B**) $8\sqrt{5}$ (**C**) $\boxed{12\sqrt{5}}$ (**D**) $16\sqrt{5}$ (**E**) $20\sqrt{5}$



6. If the diameter of a cylindrical can is increased by 30 percent, by approximating what percentage should the height be increased to triple the volume of the can?

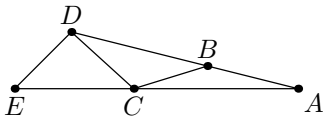
(**A**) $\boxed{77.5}$ (**B**) 54.8 (**C**) 50.0 (**D**) 44.2 (**E**) 36.8

7. Find the circumference of a circle inscribed in an isosceles right triangle with legs of 1.

(**A**) $(2+\sqrt{2})\pi$ (**B**) $(1-\sqrt{2})\pi$ (**C**) $(1+\sqrt{2})\pi$ (**D**) $\boxed{(2-\sqrt{2})\pi}$ (**E**) $(2+2\sqrt{2})\pi$

8. In the figure shown, $\angle ADE = 100^\circ$ and $DE = DC = CB = BA$. What is the measure of $\angle EAD$?

(**A**) $\boxed{20^\circ}$ (**B**) 25° (**C**) 30° (**D**) 35° (**E**) 40°

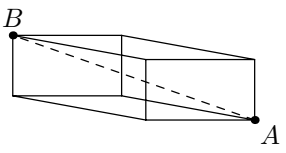


9. What is the length of the hypotenuse of a right triangle that has an area of 40 and its longest leg has length b?

(**A**) $\frac{40}{b}$ (**B**) $\frac{\sqrt{80+b^2}}{b}$ (**C**) $\frac{\sqrt{6400+b^2}}{b}$ (**D**) $\boxed{\frac{\sqrt{6400+b^4}}{b}}$ (**E**) $\sqrt{1600+b^2}$

10. In the rectangular prism shown, the area of the faces are 18, 40 and 80 respectively. What is the length of the diagonal $\overline{AB}$?

(**A**) $\boxed{\frac{\sqrt{2005}}{3}}$ (**B**) $\sqrt{138}$ (**C**) $\frac{\sqrt{79}}{3}$ (**D**) $\frac{\sqrt{2005}}{9}$ (**E**) $\sqrt{79}$



11. Find the possible values of k so that two lines $kx+y=3$ and $x-y=2$ intersect in the first quadrant.

(**A**) $k>\frac{3}{2}$ (**B**) $-1<k\leq-\frac{1}{2}$ (**C**) $k<-1$ (**D**) $2<k<3$ (**E**) $\boxed{-1<k<\frac{3}{2}}$

12. A circle is inscribed in a triangle with sides length of $4\,cm$, $13\,cm$ and $15\,cm$. Find the area of the circle.

(**A**) $1.5\pi\,cm^2$ (**B**) $\boxed{2.25\pi\,cm^2}$ (**C**) $3\pi\,cm^2$ (**D**) $3.75\pi\,cm^2$ (**E**) $4\pi\,cm^2$

13. How many faces does a polyhedral solid with 36 vertices and 64 edges have?

(**A**) $\boxed{30}$ (**B**) 48 (**C**) 56 (**D**) 64 (**E**) 86

14. A cylindrical tank has a spiral staircase one foot wide attached to its exterior. The staircase goes from the bottom to the top while making exactly 4 complete revolution if the tank is $20\,ft$ high and has a diameter of $16\,ft$. What is the length of the exterior edge of the staircase (in feet)?

(**A**) $\sqrt{100+256\pi^2}$ (**B**) $2\sqrt{100+256\pi^2}$ (**C**) $\sqrt{100+289\pi^2}$

(**D**) $\sqrt{100+324\pi^2}$ (**E**) $\boxed{4\sqrt{25+324\pi^2}}$

15. Find the radius of a circle if the difference between the area of the inscribed and circumscribed equilateral triangles is 25 square inches.

(**A**) $\frac{2}{3}\sqrt{\frac{3}{3}}$ (**B**) $\frac{4}{3}\sqrt{\frac{6}{3}}$ (**C**) $\boxed{\frac{10}{3}\sqrt{\frac{\sqrt{3}}{3}}}$ (**D**) $2\sqrt{\frac{3}{3}}$ (**E**) $4\sqrt{\frac{3}{3}}$

16. Find the area of a trapezoid in which the bases are $17\,m$ and $42\,m$ and the legs are $15\,m$ and $20\,m$.

(**A**) 225 m^2 (**B**) $\boxed{354\ m^2}$ (**C**) 400 m^2 (**D**) 475 m^2 (**E**) 554 m^2

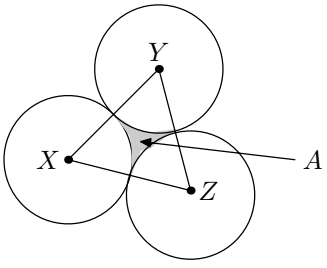
17. The sides of a triangle are the roots of $x^3-30x^2+281x-780=0$, and they are all natural numbers. What is the area of the triangle (in unit square)?

(**A**) 15 (**B**) 20 (**C**) 25 (**D**) $\boxed{30}$ (**E**) 35

18. Three circles of equal radius $\sqrt{3}$ all touch each other to enclose a three cornered concave area A, which is shaded in the figure shown. What is the area of A?

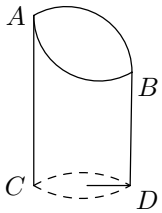
(**A**) $\sqrt{3}-\frac{\pi}{4}$ (**B**) $3\sqrt{3}-\frac{3\pi}{4}$ (**C**) $\sqrt{3}-\frac{\pi}{2}$

(**D**) $3\sqrt{3}-\frac{\pi}{4}$ (**E**) $\boxed{3\sqrt{3}-\frac{3\pi}{2}}$



19. Given the cylinder as shown, which is cut on a slant. The height goes from $15\,in$ to $21\,in$, and the radius is $4\,in$. Find the volume of the cylinder.

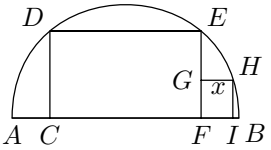
(**A**) $336\pi\,in^3$ (**B**) $\boxed{288\pi\,in^3}$ (**C**) $225\pi\,in^3$ (**D**) $144\pi\,in^3$ (**E**) $84\pi\,in^3$



20. If $ABCD$ is a square with E on DC such $DE = 2$ and $EC = 4$, then the perimeter of triangle AEB is:

(**A**) $6 + 4\sqrt{23}$ (**B**) $6 + 2\sqrt{23}$ (**C**) $6 + 4\sqrt{10} + 4\sqrt{13}$ (**D**) $\boxed{6 + 2\sqrt{10} + 2\sqrt{13}}$ (**E**) $6 + 4\sqrt{13}$

21. In the figure shown, A semicircle has diameter AB. Rectangle $CDEF$ is inscribed in the semicircle with $CD = 24$ and $DE = 56$. Square $FGHI$ with side x is between the rectangle and the semicircle as shown. What is the area of $FGHI$?

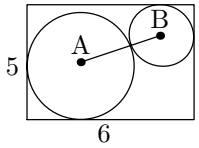


(**A**) 49 (**B**) $\boxed{64}$ (**C**) 81 (**D**) 100 (**E**) 121

22. The length, width, and the height of a rectangular solid are in the ratio of $6 : 8 : 24$. If the diagonal of the solid is $\sqrt{2704}$ inches long, what is the length of longest side of solid (in inches)?

(**A**) 12 (**B**) 24 (**C**) 36 (**D**) 40 (**E**) $\boxed{48}$

23. In the figure shown, the rectangle has a width of 6 and height of 5. Circle A has radius r and circle B has radius 1. Find the value of r.



(**A**) $\boxed{10 - \sqrt{35}}$ (**B**) $\sqrt{35} - 5$ (**C**) $12 - \sqrt{35}$ (**D**) $5 + \sqrt{35}$ (**E**) $\sqrt{35} - 6$

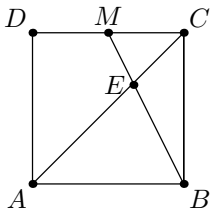
24. The center of the circle defined by $4x^2 + 4y^2 + 16x + 32y = 162$ is (h, k). What is the product of h and k?

(**A**) 6 (**B**) $\boxed{8}$ (**C**) 12 (**D**) 16 (**E**) 24

25. AB and CD are perpendicular to a diameters of circle O. Let CM be a chord that intersect AB at E, so that $CE = 6$ and $EM = 5$. Find the circumference of the circle.

(**A**) $\boxed{2\sqrt{33}\pi}$ (**B**) $\sqrt{66}\pi$ (**C**) $\sqrt{33}\pi$ (**D**) $2\sqrt{66}\pi$ (**E**) $4\sqrt{33}\pi$

26. Given the square $ABCD$ with M the midpoint of DC. The ratio of the area of the triangle MEC to that of the quadrilateral $AEMD$ is:

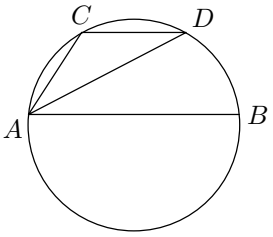


(**A**) 1 : 3 (**B**) 1 : 4 (**C**) $\boxed{1 : 5}$ (**D**) 1 : 6 (**E**) 2 : 5

27. Determine the area that is outside the square $|x| + |y| \leq 1$ and inside the circle $x^2 + y^2 = 1$.

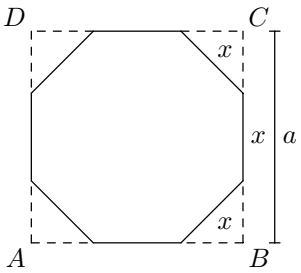
(**A**) $\pi - 3$ (**B**) $\pi - 2.5$. (**C**) $\boxed{\pi - 2}$ (**D**) $\pi - 1$ (**E**) π

28. AB is a diameter of a circle and CD is a chord parallel to AB. Find the angle $\angle C - \angle D$:



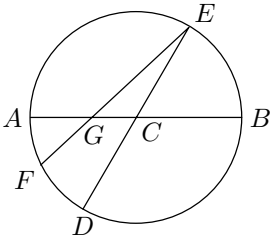
(**A**) 30° (**B**) 45° (**C**) 60° (**D**) $\boxed{90°}$ (**E**) 100°

29. The side of a square is a. Find the length of a side of a regular octagon obtained from the square by cutting off its corners.



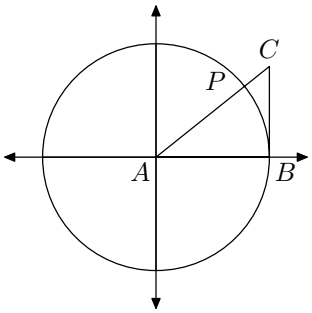
(**A**) $\frac{a}{3}$ (**B**) $\frac{a}{2}$ (**C**) $\frac{\sqrt{2}a}{2}$ (**D**) $\boxed{(\sqrt{a}-1)a}$ (**E**) $\frac{\sqrt{3}a}{3}$

30. In the figure shown, $\overline{AB}$ and $\overline{ED}$ are diameters of the given circle, intersecting on the center C of the circle. Also, F is the midpoint of the minor arc determined by points A and D, and the chord $\overline{EF}$ intersects $\overline{AB}$ on the point G. If $\angle BCE$ has measure 60°, then the measure of $\angle AGF$ is:



(**A**) 15° (**B**) 30° (**C**) $\boxed{45°}$ (**D**) 60° (**E**) 75°

31. In the figure shown, $\overline{BC}$ is tangent to the circle centered at the origin. If $\overline{AC}$ intersects the circle at $P(\frac{\sqrt{3}}{2}, \frac{1}{2})$, then the length of the $\overline{PC}$ is:

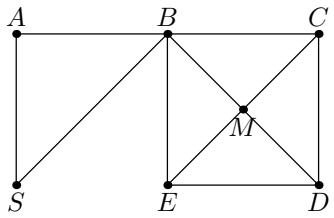


(**A**) $\frac{2}{\sqrt{3}}$ (**B**) $\sqrt{3}-1$ (**C**) $\boxed{\frac{2\sqrt{3}-3}{3}}$ (**D**) 1 (**E**) $\frac{3}{2}$

32. Sam sees that the top of a 15 foot lamp, which is 250 feet away, lines up perfectly with the peak of a distant mountain. Sam knows that the mountain is 15 miles away so he uses the lamp to determine the height of the mountain. If Sam's eyes are 5 feet above the ground, what is the best estimate of the height of the mountain relative to Sam?

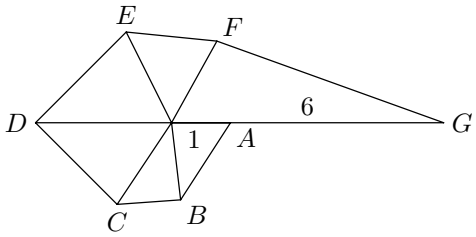
(**A**) 900 feet (**B**) $\boxed{3178\text{ feet}}$ (**C**) 4752 feet (**D**) 5625 feet (**E**) 6648 feet

33. In the figure shown, how many different paths are there from S to E if one never visits the same point twice?



(**A**) less than 11 (**B**) 11 (**C**) 12 (**D**) 13 (**E**) $\boxed{\text{more than 13}}$

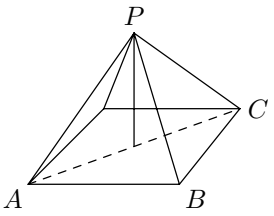
34. In the figure shown, six similar triangles are each sharing one side with the next triangle and all are sharing one vertex. All angles at that vertex measure 60°. If the side of the last (*smallest*) triangle that is adjoining the first triangle is 1/6 as large as the longest side of the first triangle, how many times larger is the area of the largest triangle as compared to the smallest?



(**A**) 6 (**B**) $\sqrt[6]{6^5}$ (**C**) $\boxed{\sqrt[3]{6^5}}$

(**D**) 36 (**E**) none of these

35. Find the height of a square pyramid formed by four equilateral triangles whose sides all have length 2.



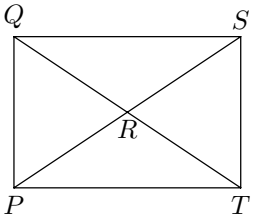
(**A**) 1 (**B**) $\frac{\sqrt{6}}{2}$ (**C**) $\boxed{\sqrt{2}}$ (**D**) $\sqrt{3}$ (**E**) 2

36. In triangle ABC, the measure of angle A is twice the measure of angle B and the measure of angle B is three times the measure of angle C. The measure of angle B is:

(**A**) $\boxed{\text{less than } 60^\circ}$ (**B**) at least 60° but less than 65°

(**C**) at least 65° but less than 70° (**D**) at least 70° but less than 75° (**E**) at least 75°

37. In the figure shown, the area of rectangle $PQST$ is 9 square inches. If the degree measure of $\angle PRQ$ is 60°, the length of side PQ (in inches) is:



(**A**) $\sqrt[4]{3}$ (**B**) $\sqrt{3}$ (**C**) $\boxed{\sqrt[4]{3^3}}$ (**D**) 3 (**E**) $\sqrt[4]{3^5}$

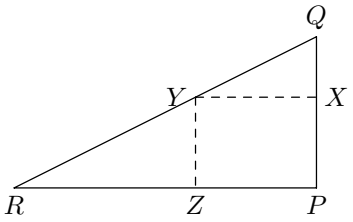
38. Let A be the point $(3, 2)$, and B be the reflection of A about the x-axis. Let C be the reflection point of B about the line $y = -x$ and D be the reflection point of C about the origin. what is the area of the quadrilateral $ABCD$?

(**A**) 14 (**B**) $\boxed{15}$ (**C**) 16 (**D**) 17 (**E**) 18

39. A and B are two points on the circle of radius r and center O. If the distance between A and B is r, what is the radian measure of $\angle AOB$?

(**A**) $\frac{\pi}{6}$ (**B**) $\frac{\pi}{4}$ (**C**) 1 (**D**) $\boxed{\frac{\pi}{3}}$ (**E**) $\frac{\pi}{2}$

40. In the figure shown, PQR is a $3-4-5$ right triangle with right angle at P. Points X, Y, Z on PQ, QR, PR respectively are chosen so that $PXYZ$ is a square. What is the length of the segment PX?



(**A**) $\frac{\sqrt{3}}{2}$ (**B**) $\frac{2}{\sqrt{3}}$ (**C**) $\frac{5}{4}$ (**D**) $\frac{5}{3}$ (**E**) $\boxed{\frac{12}{7}}$

41. Given a right triangle with right angle at point C and legs of length a and b, the length of the segment which joins C to the hypotenuse of the triangle and which bisects the angle at C is given by the formula:

(**A**) $\frac{ab}{\sqrt{a^2+b^2}}$ (**B**) $\frac{\sqrt{ab}}{2}$ (**C**) $\frac{a+b}{2\sqrt{2}}$ (**D**) $\frac{2ab}{\sqrt{a^2+b^2}}$ (**E**) $\boxed{\frac{\sqrt{2}ab}{a+b}}$

42. Equilateral triangle ABC has vertices A, B and C, and center D. A new triangle $A'B'C'$ is formed where A' is the midpoint of AD, B' is the midpoint of BD, and C' is the midpoint of CD. The ratio of the area of triangle ABC to the triangle $A'B'C'$ is:

(**A**) 3 : 1 (**B**) 3 : 2 (**C**) 9 : 4 (**D**) 2 : 1 (**E**) $\boxed{4:1}$

43. Let C be the circle described by $(a-x)^2+y^2=r^2$ where $0<r<a$. Let m be the slope of the line through the origin that is tangent to C at a point in the first quadrant. Then

(**A**) $\boxed{m=\frac{r}{\sqrt{a^2-r^2}}}$ (**B**) $m=\frac{\sqrt{a^2-r^2}}{r}$ (**C**) $m=\frac{r}{a}$ (**D**) $m=\frac{a}{r}$ (**E**) $m=\frac{a^2}{r^2}-1$

44. In $\triangle ABC$, $\overline{BC}=24$, $\overline{AC}=18$ and the medians to sides $\overline{BC}$ and $\overline{AC}$ are perpendicular. Find $\overline{AB}$.

(**A**) 30 (**B**) $15\sqrt{2}$ (**C**) $\boxed{6\sqrt{5}}$ (**D**) $5\sqrt{6}$ (**E**) none of these

45. In a three dimensional rectangular coordinate system, find the total surface area of the solid defined by $|x|+|y|+|z|\leq 1$.

(**A**) $4\sqrt{2}$ (**B**) $2\sqrt{6}$ (**C**) $\boxed{4\sqrt{3}}$ (**D**) $2\sqrt{5}$ (**E**) $3\sqrt{2}$

46. If the graphs of $2y+x+3=0$ and $3y+ax+2=0$ are to meet at right angle, then a is

(**A**) $\boxed{-6}$ (**B**) 6 (**C**) $-\frac{2}{3}$ (**D**) $\frac{3}{2}$ (**E**) none of these

47. A pair of opposite vertices and the midpoints of a pair of opposite edges of a cube are connected to form a quadrilateral. If each edge of the cube has length k, find the area of the quadrilateral.

(**A**) $\frac{\sqrt{3}}{2}k^2$ (**B**) $\boxed{\frac{\sqrt{6}}{2}k^2}$ (**C**) $\frac{\sqrt{2}}{3}k^2$ (**D**) $\frac{k^2}{2}$ (**E**) $\frac{\sqrt{6}}{3}k^2$

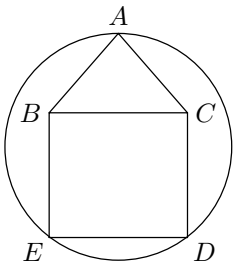
48. You have 6 sticks of lengths 10, 20, 30, 40, 50 and 60 centimeters. The number of non-congruent triangles that can be formed by choosing three of the sticks to make the sides is

(**A**) 3 (**B**) 6 (**C**) $\boxed{7}$ (**D**) 10 (**E**) 12

49. A glass box of $7\,cm\times 12\,cm\times 18\,cm$, closed on all six sides, is partly filled with colored water. When the box is placed on one of its 7×12 sides, the water level is $15\,cm$ above the table. If the box is placed on one of its 7×18 sides, what is the water level above the table, in centimeters?

(**A**) 7.5 (**B**) 9 (**C**) $\boxed{10}$ (**D**) 10.5 (**E**) 11

50. In the figure shown, $ABEDC$ is circumscribe a circle through points A, D and E. ABC is an equilateral triangle with side length 2, and $BCDE$ is a square. Find the radius of the circle.



(**A**) $\frac{\sqrt{3}}{2}+1$ (**B**) $\boxed{2}$ (**C**) $\sqrt{3}+1$ (**D**) $5-2\sqrt{3}$ (**E**) $\sqrt{2}$