

2010

ALABAMA

STATEWIDE MATHEMATICS CONTEST



First Round : March 27, 2010
 Second Round: April 24, 2010 at The University of Alabama

GEOMETRY EXAMINATION

Construction of this test directed
 by
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 and
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INSTRUCTIONS

This test consists of 50 multiple choice questions. The questions have not been arranged in order of difficulty. For each question, choose the best of the five answer choices labeled A, B, C, D, and E.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered, and 0 points for each wrong answer. (Thus a “perfect paper” with all questions answered correctly earns a score of 250, a blank paper earns a score of 50, and a paper with all questions answered incorrectly earns a score of 0.)

Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate one or more of the answer choices as wrong, then it is to your advantage to guess among the remaining choices.

- All variables and constants, except those indicated otherwise, represent real numbers.
- Diagrams are not necessarily to scale.

We use the following geometric notation:

- | | |
|--|--|
| • If A and B are points, then: | • If A is an angle, then: |
| \overline{AB} is the segment between A and B | $m \angle A$ is the measure of angle A in degrees |
| \overleftrightarrow{AB} is the line containing A and B | • If A and B are points on a circle, then: |
| \overrightarrow{AB} is the ray from A through B | \widehat{AB} is the arc between A and B |
| AB is the distance between A and B | $m \widehat{AB}$ is the measure of \widehat{AB} in degrees |

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1. The angle of a sector is 96° and its arc is 24 in . Find the area of the circle (in square inches).

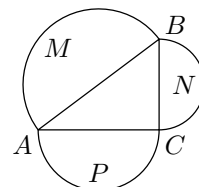
(A) $\frac{2025}{\pi}$ (B) $\frac{1800}{\pi}$ (C) $\frac{900}{\pi}$ (D) $\frac{450}{\pi}$ (E) $\frac{225}{\pi}$

2. In the triangle ABC , $AB = 16 \text{ in}$, $AC = 9 \text{ in}$ and the median from $\angle C$ is 11 in . Find the perimeter of the triangle.

(A) 40 in (B) 42 in (C) 44 in (D) 46 in (E) 48 in

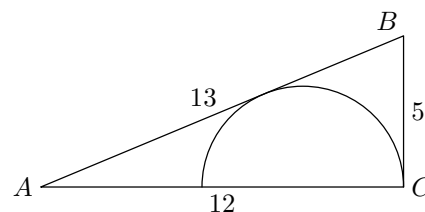
3. In the figure shown, each side of right triangle ABC is the diameter of a semicircle. The area of semicircle M is 100π and the area of semicircle N is 36π . Find the perimeter of $\triangle ABC$.

(A) $100\sqrt{2}$ (B) $72\sqrt{2}$ (C) $64\sqrt{2}$ (D) $56\sqrt{2}$ (E) $48\sqrt{2}$



4. In the right triangle ABC with legs of length 5 and 12 and the hypotenuse of length 13. A semicircle is inscribed in the triangle. Find the radius of the semicircle.

(A) $\frac{17}{3}$ (B) $\frac{13}{5}$ (C) $\frac{10}{3}$ (D) $\frac{12}{5}$ (E) $\frac{7}{3}$

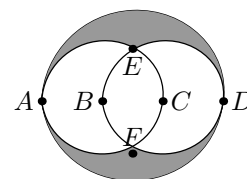


5. The area of a right triangle is 64. What is the shortest possible length of the hypotenuse?

(A) $6\sqrt{6}$ (B) 16 (C) 12 (D) $4\sqrt{3}$ (E) 6

6. In the figure shown, two small circles are inscribed and tangent to the large circle at points A and D . Let points B and C be the centers of the circles, where $AB = BC = CD = 1$. Find the area of the shaded region.

(A) $\frac{5\pi}{12} - \frac{\sqrt{3}}{2}$ (B) $\frac{13\pi}{24} - \frac{\sqrt{3}}{2}$ (C) $\frac{7\pi}{12} - \frac{\sqrt{3}}{2}$
 (D) $\frac{11\pi}{24} - \frac{\sqrt{3}}{2}$ (E) $\frac{11\pi}{12} - \frac{\sqrt{3}}{2}$



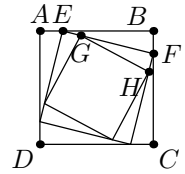
7. The interior angles of a polygon are in arithmetic progression; the least angle is 120° and the common difference is 5° . Find the greatest number of sides that this polygon can have.

(A) 20 (B) 16 (C) 12 (D) 9 (E) 7

8. Determine the equation of the line passing through the points of intersection of the circles $(x + 2)^2 + (y + 1)^2 = 9$ and $(x + 3)^2 + (y + 4)^2 = 16$.

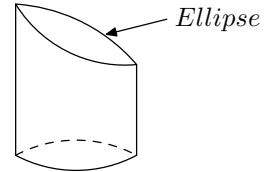
(A) $2x + 6y = -13$ (B) $10x - 6y = -7$
 (C) $-2x + 6y = -7$ (D) $-10x + 6y = 25$ (E) $2x - 6y = 13$

9. You have three inscribed squares, with corners of each square at the $\frac{1}{5}$ point along the sides of its outer square. The area of the largest square is 625 in^2 . What is the area of the smallest square?



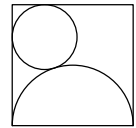
- (A) 109 in^2 (B) 125.2 in^2 (C) 225 in^2 (D) 289 in^2 (E) 325.2 in^2

10. A cylinder is sliced by a plane to form the figure shown. The base edge of the figure is a circle of radius 4 in . The top edge is an ellipse. The highest point on the ellipse is 10 in above the base and the lowest point on the ellipse is 6 in above the base. Find surface area of this figure.



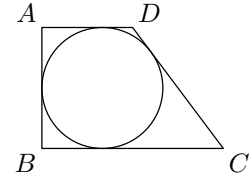
- (A) $32\pi \text{ in}^2$ (B) $64\pi \text{ in}^2$ (C) $72\pi \text{ in}^2$ (D) $90\pi \text{ in}^2$ (E) $96\pi \text{ in}^2$

11. In the diagram shown, a semicircle and a circle are placed inside a square with side length of 4. Determine the radius of the circle.



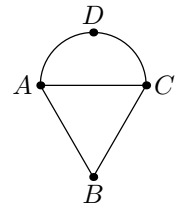
- (A) $1 + \sqrt{3}$ (B) $6 - 2\sqrt{3}$ (C) $8 - 4\sqrt{3}$ (D) $2 + \sqrt{3}$ (E) $3 + \sqrt{3}$

12. In the trapezoid $ABCD$, where $m\angle DAB = 90^\circ$, a circle is inscribed as shown. The circle's point of tangency divides line segment \overline{CD} into lengths of 4 and 16. Find the area of the trapezoid.



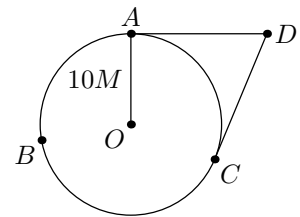
- (A) 576 (B) 288 (C) 144 (D) 72 (E) 36

13. An equilateral triangle ABC and a semicircle are attached in the figure shown. Let point D be halfway along the arc of the semicircle. If $BD = 2\sqrt{3}$, determine the length of the arc \widehat{AC} .



- (A) $(3 - \sqrt{3})\pi$ (B) $(5 + \sqrt{3})\pi$
 (C) $(7 - 2\sqrt{3})\pi$ (D) $(3 + \sqrt{3})\pi$ (E) $(5 - \sqrt{3})\pi$

14. Two tangents to a circle form an angle equal to 50° . If the radius of the circle is $10M$, determine the length of the smaller of the intercepted arcs.



- (A) $\frac{19}{9}\pi M$ (B) $\frac{25}{9}\pi M$ (C) $\frac{44}{9}\pi M$ (D) $\frac{52}{9}\pi M$ (E) $\frac{65}{9}\pi M$

15. A regular hexagon has sides 8 in long. Find the area of the triangle formed by connecting alternate vertices.

- (A) $36\sqrt{3} \text{ in}^2$ (B) $48\sqrt{3} \text{ in}^2$ (C) $54\sqrt{3} \text{ in}^2$ (D) $60\sqrt{3} \text{ in}^2$ (E) $72\sqrt{3} \text{ in}^2$

16. What the exact area of the triangle whose vertices are $(0, 2)$, $(1, 4)$ and $(2, 3)$?

- (A) 3 (B) $2\frac{1}{2}$ (C) $1\frac{1}{2}$ (D) 1 (E) $\frac{1}{2}$

17. In the $\triangle ABC$, \overline{OB} bisects $\angle ABC$, and $AB = AC$. If $m\angle ABO = 2x + 20$ and $m\angle CBO = 4x + 12$, what is $m\angle BAC$?

(A) 90° (B) 68° (C) 56° (D) 38° (E) 26°

18. A chord is perpendicular to a diameter of a circle at a point which divides the diameter into segments having the ratio 1 : 3. In what ratio does the chord divide the circumference?

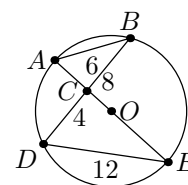
(A) 1 : 4 (B) 2 : 4 (C) 3 : 5 (D) $1 : 2$ (E) 3 : 4

19. Hannah bought a box in the shape of a rectangular prism. She found the lengths of the diagonals of the faces to be $20, 2\sqrt{61}, 2\sqrt{89}$. Determine the volume of the box.

(A) $1920 U^2$ (B) $1440 U^2$ (C) $960 U^2$ (D) $480 U^2$ (E) $320 U^2$

20. In the figure shown, four chords are drawn so that two intersect at point C and chord \overline{AE} is the diameter of the circle. If $AB = 6, BC = 8, CD = 4$ and $DE = 12$, then what is the area of the circle?

(A) $27U^2$ (B) $64U^2$ (C) $81U^2$ (D) $96U^2$ (E) $108U^2$



21. If six golf balls are packed tightly in a cylindrical can, then what fraction of the can is unoccupied?

(A) $\frac{1}{6}$ (B) $\frac{2}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) $\frac{3}{4}$

22. In value, the area of an equilateral triangle is equal to its perimeter. What is the length of the side of the triangle?

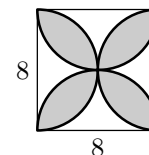
(A) $4\sqrt{3} U$ (B) $2\sqrt{3} U$ (C) $6\sqrt{3} U$ (D) $\sqrt{3} U$ (E) $3\sqrt{3} U$

23. Find the area of a trapezoid in which the bases are $17 FT$ and $42 FT$ and the legs are $15 FT$, and $20 FT$.

(A) $340 FT^2$ (B) $354 FT^2$ (C) $455 FT^2$ (D) $595 FT^2$ (E) $708 FT^2$

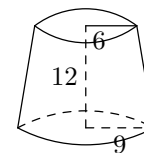
24. Find the area of the shaded region shown in the figure where the petals are formed by the semicircles.

(A) $32\pi - 64$ (B) $32\pi - 32$ (C) $32\pi - 16$ (D) $2\pi - 8$ (E) $2\pi - 4$



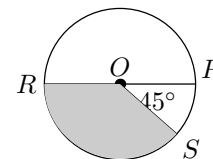
25. The volume of the truncated cone in the figure shown is

(A) 36 (B) 64 (C) 81 (D) 144 (E) None of these



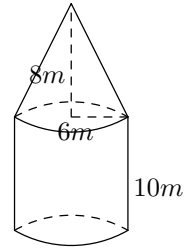
26. What is the area of the shaded region shown in the figure if the diameter \overline{RP} of the circle is 8?

(A) 1.5π (B) 6π (C) 24π (D) 72π (E) 100π



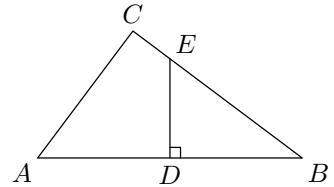
27. Rounded to the nearest hundred cubic meters, what is the total capacity (cone and cylinder) of the storage container shown in the figure?

(A) $\boxed{1,400}$ (B) 2,000 (C) 5,700 (D) 8,100 (E) 9,400



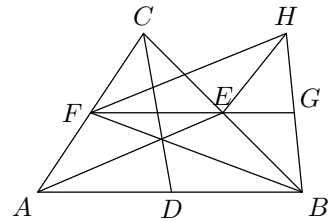
28. In the figure shown, $m\angle C = 90^\circ$, $AD = DB$, $\overline{DE} \perp \overline{AB}$, $AB = 20$ and $AC = 12$. Find the area of the quadrilateral $ADEC$.

(A) 75 (B) $\boxed{58\frac{1}{2}}$ (C) $37\frac{1}{2}$ (D) 48 (E) 36



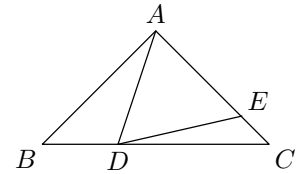
29. In the triangle ABC , AE, BF, CD are medians. FH is parallel and equal in length to AE . BH and HE are drawn, and FE is extended and meets BH in G . Which one of the following statement is not necessarily correct?

(A) $AEHF$ is parallelogram (B) $\boxed{HE = HG}$ (C) $BH = DC$
 (D) $FG = \frac{3}{4}AB$ (E) FG is a median of triangle BFH



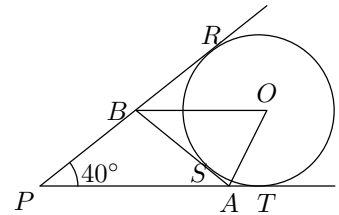
30. In the figure shown, $AB = AC$, $m\angle BAD = 30^\circ$ and $AE = AD$. Find x (which is $m\angle EDC$).

(A) 7.5° (B) 10° (C) $\boxed{15^\circ}$ (D) 12.5° (E) 20°



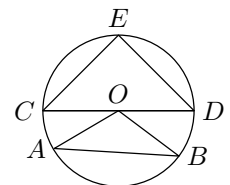
31. In the figure shown, $m\angle P = 40^\circ$ and $\triangle PAB$ is formed by three tangents to the circle O . Find $m\angle AOB$.

(A) 45° (B) 50° (C) 55° (D) 160° (E) $\boxed{70^\circ}$



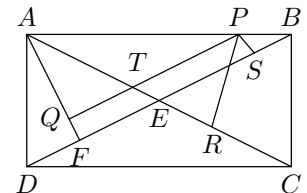
32. In the figure shown, \overline{CE} and \overline{DE} are equal chords of a circle with center O . Arc \widehat{AB} is a quarter of the circle. Find the ration of the area of $\triangle CED$ to the area of $\triangle AOB$.

(A) $\sqrt{2} : 1$ (B) $\sqrt{3} : 1$ (C) $4 : 1$ (D) $3 : 1$ (E) $\boxed{2 : 1}$



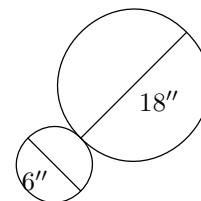
33. In the figure shown, $ABCD$ is a rectangle with P , a point on \overline{AB} , and $\overline{PS} \perp \overline{BD}$, $\overline{PR} \perp \overline{AC}$, $\overline{AF} \perp \overline{BD}$ and $\overline{PQ} \perp \overline{AF}$. What is the sum $PR + PS$ equal to?

(A) PQ (B) AE (C) $PT + AT$ (D) \boxed{AF} (E) EF



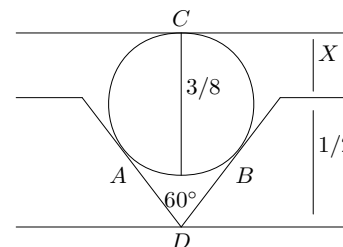
34. Two poles, 6-inch and 18-inch in diameters, are placed, as in the figure shown, and bounded together with wire. The length of the shortest wire that will go around them is

- (A) $12\sqrt{3} + 16\pi$ (B) $12\sqrt{3} + 7\pi$
 (C) $12\sqrt{3} + 14\pi$ (D) $12\sqrt{12} + 15\pi$ (E) 24π



35. In the figure shown, A, B, C are points of tangency to the circle with diameter $\frac{3}{8}$ inches. Find X (in inches).

- (A) $\frac{3}{16}$ (B) $\frac{1}{8}$ (C) $\frac{1}{32}$ (D) $\frac{3}{32}$ (E) $\frac{1}{16}$

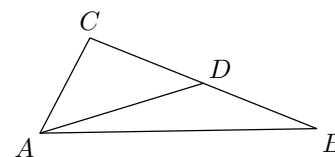


36. On a map whose scale is 400 miles to an inch and a half, a certain estate is represented by a rhombus having a 60° angle. The diagonal opposite 60° is $\frac{3}{16}$ in. What is the area of the estate in square miles?

- (A) $\frac{2500}{\sqrt{3}}$ (B) $\frac{1250}{\sqrt{3}}$ (C) 1250 (D) $\frac{5625\sqrt{3}}{2}$ (E) $1250\sqrt{3}$

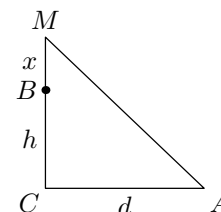
37. In the $\triangle ABC$, $\overline{AC} = \overline{CD}$ and $m\angle CAB - m\angle ABC = 30^\circ$. Find $m\angle BAD$.

- (A) 30° (B) 10° (C) $22\frac{1}{2}^\circ$ (D) 15° (E) 20°



38. In the right triangle shown, the sum of distances BM and MA is equal to the sum of distances BC and CA . If $MB = x$, $CB = h$ and $CA = d$, then x equals

- (A) $\frac{hd}{2h+d}$ (B) $d-h$ (C) $\frac{1}{2}d$ (D) $h+d-\sqrt{2d}$ (E) $\sqrt{h^2+d^2}-h$



39. In the triangle ABC , $\overline{AB} = \overline{AC}$, $m\angle A = 40^\circ$ and point O is within the triangle with $\angle OBC = \angle OCA$. Find $m\angle BOC$.

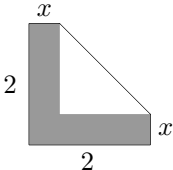
- (A) 140° (B) 35° (C) 110° (D) 55° (E) 70°

40. In the triangle ABC , $AC = 12$. If one of the trisectors of angle B is the median to \overline{AC} and the other trisector of angle B is the altitude to \overline{AC} , find the length of the altitude.

- (A) $2\sqrt{3}$ (B) $3\sqrt{3}$ (C) $4\sqrt{3}$ (D) $7\sqrt{5}$ (E) $\sqrt{3}$

41. At what fraction of an hour after 3 o'clock are the minute and hour hands of a twelve-hour clock pointing in the same direction?

- (A) $\frac{3}{4}$ (B) $\frac{3}{7}$ (C) $\frac{3}{11}$ (D) $\frac{3}{17}$ (E) $\frac{3}{19}$

42. What's the area of a circle inside of which is inscribed a triangle with side lengths 5, 12, 13?
 (A) 32.25π (B) $\boxed{42.25\pi}$ (C) 44.25π (D) 50.25π (E) 60π
43. Circle B passes through the center of circle A and is tangent to it. Circle C passes through the center of Circle B and is tangent to it. What fraction of the area of Circle A lies inside Circle B but outside C ?
 (A) $\frac{1}{4}$ (B) $\frac{1}{8}$ (C) $\frac{1}{16}$ (D) $\boxed{\frac{3}{16}}$ (E) $\frac{17}{32}$
44. A rhombus has a 60 degree angle. What is the ratio of its area to that of a circle inscribed inside it?
 (A) $\frac{7\sqrt{3}}{3\pi}$ (B) $\frac{7\sqrt{2}}{3\pi}$ (C) $\boxed{\frac{8\sqrt{3}}{3\pi}}$ (D) $4\sqrt{3}\pi$ (E) 5π
45. In the triangle ABC , angle A is 120 degrees, $BC + AB = 21$ and $BC + AC = 20$. What's the sum of the three sides?
 (A) $\boxed{28}$ (B) 21 (C) 24 (D) 13 (E) 20
46. In the triangle ABC , $BC = 2\sqrt{2}$, $CA = 12$ and $m\angle ACB = 135^\circ$. If D is the midpoint of AB , what is the length of CD ?
 (A) $\frac{2\sqrt{26}}{3}$ (B) $\sqrt{21}$ (C) $\frac{2\sqrt{21}}{3}$ (D) $\boxed{\sqrt{26}}$ (E) $2\sqrt{29}$
47. In the figure shown, if the area of the letter "L" equals the area of the triangle, what is the length x of the ends of the L ? The right answer is $\frac{6 - 2\sqrt{6}}{3}$.
 (A) $\frac{3 + \sqrt{13}}{3}$ (B) $\frac{3 - \sqrt{13}}{3}$ (C) $\frac{5 + \sqrt{13}}{3}$ (D) $\frac{5 - \sqrt{13}}{3}$ (E) $\sqrt{13}$
- 
48. What is the maximum number of points with integer coordinates which can lie on a circle with center at $(\sqrt{3}, \sqrt{5})$?
 (A) 0 (B) 3 (C) 2 (D) $\boxed{1}$ (E) 4
49. A cube and sphere have the same surface area. What is the ration of volume of the sphere to that of the cube?
 (A) $\sqrt{\frac{5}{\pi}}$ (B) $\frac{2}{\pi}$ (C) $\frac{3}{\pi}$ (D) $\boxed{\sqrt{\frac{6}{\pi}}}$ (E) 1
50. What is the radius of a circle inscribed in an isosceles right triangle whose legs have length 2?
 (A) $\sqrt{2}$ (B) 2 (C) $2\sqrt{3}$ (D) $2 - \sqrt{3}$ (E) $\boxed{2 - \sqrt{2}}$