Auburn University Math-a-Thon

January 26, 2008

Calculators are NOT allowed!

Circle the letter for the correct answer on the answer sheet.

1. A rectangle is inscribed in a circle of radius 3. A quadrilateral is formed by connecting the midpoints of each of the sides of the rectangle. What is the perimeter of this quadrilateral?

   a) 3 b) 6 c) 9 d) 12 e) 15

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2. Consider the regular pentagon Pent($ABCDE$) with the vertices listed in order clockwise. Calculate the measure of $\angle BAC$.

   a) 12° b) 36° c) 60° d) 84° e) 108°

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3. An equilateral triangle with a side of length 5 is inscribed in a circle. What is the radius of the circle?

   a) 2.5 b) $\frac{5\sqrt{3}}{3}$ c) $\frac{\sqrt{3}}{5}$ d) $\frac{5\sqrt{2}}{2}$ e) $\frac{\sqrt{2}}{5}$

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4. Given:

   $$x = \frac{\frac{a}{b}}{c}; \qquad y = \frac{a}{\frac{b}{c}} \qquad \text{find} \qquad \frac{x}{y}.$$

   a) $\frac{1}{a}$ b) $\frac{1}{c}$ c) $\frac{c}{b}$ d) $\frac{1}{a^2}$ e) $\frac{1}{c^2}$

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5. A square based pyramid on a 12 × 12 square has a height of 9. What is the area of the cross-sectional square 3 units up from the base.

   a) 25 b) 36 c) 49 d) 64 e) 81

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6. As a fraction in lowest terms $0.13192\overline{29} = ?$

   a) $\frac{46}{343}$ b) $\frac{653}{4950}$ c) $\frac{13061}{99000}$ d) $\frac{23}{175}$ e) $\frac{1457}{11044}$

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7. Find the length of the diagonal of the cube that has an edge length of $\sqrt{5}$.

   a) $\sqrt{5}$ b) $\sqrt{10}$ c) $\sqrt{15}$ d) 5 e) $5\sqrt{5}$

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8. If $\log_7 5 = A$ then what is $\log_5 7$?

a) $-A$ b) $-\dfrac{1}{A}$ c) $\dfrac{1}{A}$ d) A^5 e) A^7

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9. If $\log_2(\log_3 x) = 2$ then $x =$?

a) 81 b) 49 c) 36 d) 4 e) 3

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10. How many zeros are there to the right of the last right-most non-zero digit of the integer 100!?

a) 11 b) 17 c) 19 d) 21 e) 24

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11. What is the angle between the hour and minute hands at 5:15 on a standard 12-hour clock?

a) 58.5° b) 60° c) 62.5° d) 65° e) 67.5°

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12. An equilateral triangle has a height of 4. What is the length of the side of the largest inscribed square?

a) $\dfrac{4}{\sqrt{3}}$ b) 2 c) $\dfrac{8}{3}$ d) $\dfrac{8}{2+\sqrt{3}}$ e) $2\sqrt{3}$

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13. A plane cuts the cube of edge length 3 in a regular hexagon; what is the length of the side of the hexagon?

a) $\dfrac{3}{2}\sqrt{2}$ b) $\dfrac{2}{3}\sqrt{3}$ c) $\dfrac{\sqrt{5}-1}{2}$ d) $\dfrac{2}{3}\sqrt{5}$ e) $\dfrac{\sqrt{6}-1}{2}$

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14. What is the length of the side of the largest equilateral triangle inscribed in a square of side 3 units?

a) 3 b) $\sqrt{6-\sqrt{2}}$ c) $\sqrt{6}-1$ d) $3(\sqrt{6}-\sqrt{2})$ e) $3\sqrt{2}-\dfrac{1}{\sqrt{3}}$

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15. Evaluate

$$\sqrt{6+\sqrt{6+\sqrt{6+\cdots}}}$$

a) -2 b) -1 c) 1 d) 2 e) 3

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16. Suppose that there are three circles (all of different sizes), centered at the points A, B, and C, with each circle tangent to the other two (on the outside). Given that the lengths of the segments AB, AC, and BC are 3, 4, and 5 respectively, what is the radius of the largest of the three circles?

a) 2 b) 3 c) 7/2 d) 4 e) 5

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17. A person has fifty 2-dollar bills, twenty 5-dollar bills, and no other money of any kind. What is the largest integer number of dollars between 1 and 100 which the person can NOT pay using exact change?

a) 3 b) 9 c) 13 d) 14 e) None of the above

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18. Let A, B, and C be three equally spaced points on a circle (that is, they form an equilateral triangle). Let D be a randomly chosen point on the shorter arc from A to B (with D different from both A and B). What is the measure of the angle ADB?

a) 90° b) 120° c) 135° d) 150° e) Not enough information

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19. What is the last digit (in base 10) of 3^{2008}?

a) 1 b) 3 c) 5 d) 7 e) 9

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20. Along a long hallway there are 1000 classrooms. The first classroom is packed with 1000 students. The rest of the classrooms are empty. After each minute at the sound of a whistle from each classroom where more than one student stayed, one student goes over to the next classroom. How many classrooms are empty after one hour?

a) 931 b) 939 c) 969 d) 940 e) 960

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21. What is the largest exponent n so that 8^n is a divisor of 44^{44}?

a) 29 b) 44 c) 13 d) 8 e) 88

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22. If 12 people complete a certain job in 5 days, then how many days are needed for 15 people to complete half of the job?

a) 1 b) 2 c) 3 d) 4 e) 5

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23. The least common multiple of two integers is 48. The largest common divisor of the same two integers is 8. What is the sum of the digits of the product of the two integers?

a) 10 b) 15 c) 20 d) 25 e) 30

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24. What is the number of those four digit numbers, which if read from right to left also give four digit numbers?

a) 9000 b) 8991 c) 8910 d) 8100 e) 7963

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25. If m and n are integers such that $m - 5n = 4$, then what will $5m - n$ equal?

a) -4 only b) 0 only c) only multiples of 4 d) any integer e) none of these

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26. How many positive integers are there so that the product of their digits is 10 and the sum of their digits is 9?

a) 12 b) 18 c) 25 d) 36 e) 45

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27. Find the number of those positive integers who representations both in the base 5 and in the base 6 systems consist of exactly four digits.

a) 623 b) 409 c) 398 d) 389 e) 420

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28. Among the integers 11, 22, 33, 44, 55, 66, 77, 88, and 99, how many can be the last two digits of a square of an integer?

a) 2 b) 3 c) More than three d) 0 e) 1

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29. Arrange the given three polygons according to their areas from the smallest to the largest: the square S of diagonal 2, the triangle T with sides $\frac{3}{2}$, 2, and $\frac{5}{2}$, and the regular hexagon H of diagonal $\sqrt{3}$.

a) $S < H < T$ b) $T < S < H$ c) $T < H < S$ d) $H < T < S$ e) $H < S < T$

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30. Four vertices A, B, C, and D of the unit cube are selected so that no two of them lie on the same edge of the cube. Find the volume of the tetrahedron $ABCD$.

a) $\frac{1}{6}$ b) $\frac{1}{4}$ c) $\frac{1}{3}$ d) $\frac{\sqrt{3}}{2}$ e) $\frac{\sqrt{2}}{3}$

Answers

1. d	2. b	3. b	4. e	5. d	6. c	7. c	8. c	9. a	10. e
11. e	12. d	13. a	14. d	15. e	16. b	17. a	18. b	19. a	20. c
21. a	22. b	23. b	24. d	25. c	26. a	27. b	28. e	29. c	30. c