

**2000 Hoover High School Mathematics Tournament**  
**Algebra I Ciphering**

1.1 If  $a$  is the  $x$ -intercept,  $b$  the  $y$ -intercept, and  $c$  the slope of the line  $4x + 5y = 20$ , then find the product  $abc$ .

**Answer:** -16.

1.2 Twenty marbles are in a bag. There are 5 green marbles, 9 blue marbles, and the rest are red. Two marbles are drawn at random without replacement. Find the probability that neither is blue.

**Answer:**  $11/38$ .

1.3 Simplify the following expression when  $a = 3$  and  $b = -\frac{1}{4}$ .

$$\left(\frac{a^3 + b^3}{a - b}\right)\left(\frac{a^2 - b^2}{a^2 - ab + b^2}\right)\left(\frac{4}{a + b}\right)$$

**Answer:** 11.

1.4 A square is converted to a rectangle by increasing its height by 5 and decreasing its width by 5. By how much is the area decreased?

**Answer:** 25.

1.5 What is the degree of the following equation?

$$x^4(1 - x^3) + x^6 = x^2 + 2x^5 - x^7$$

**Answer:** 6.

2.1 A rectangular box has length 3, height 4, and width 5. Find the length of its space diagonal.

**Answer:**  $5\sqrt{2}$ .

2.2 Simplify completely:

$$\frac{\left(\frac{x - y}{x + y}\right)\left(\frac{1}{x} + \frac{1}{y}\right)}{\left(\frac{1}{x} - \frac{1}{y}\right)}$$

**Answer:** -1.

2.3 In two years, Ren will be twice as old as Stimpy was four years ago. If Ren is five years older than Stimpy, then how old is Ren now?

**Answer:** 20.

2.4 Find the value of  $x + y$  if  $4x + 3y = 7$  and  $3x + 4y = 14$ .

**Answer:** 3.

2.5 Solve the following equation for  $c$ :

$$4c^2 + 25 = (c + 3)^2 + 20c$$

**Answer:** 8,  $2/3$ .

3.1 If A is 20% of B, C is 130% of D, and C is 75% of B, then what is the ratio D:A?

**Answer:** 75:26.

3.2 Consider the set of numbers  $\{1, 2, 3, \dots, 12, 13\}$ . If  $m$  is the mean of this set and  $d$  is the median of the set, then find  $m + d$ .

**Answer:** 14.

3.3 Find the sum of the roots of the equation  $3x^2 - 5x = 2$ .

**Answer:**  $5/3$ .

3.4 What is the length of the line  $3x + 5y = 15$  that lies in the first quadrant?

**Answer:**  $\sqrt{34}$ .

3.5 If the difference of the areas of two circles is  $48\pi$  and the sum of their radii is 16, then what is the positive difference of their circumferences?

**Answer:**  $6\pi$ .

4.1 Find the sum of the first 10 positive odd numbers.

**Answer:** 100.

4.2 Solve the equation for  $k$ :

$$k^2 + k - 156 = 0$$

**Answer:** 12, -13.

4.3 Simplify completely:

$$\frac{\left(\frac{(x+1)^2}{2x+1} - 1\right)^2}{x^3}$$

**Answer:**  $\frac{x}{(2x+1)^2}$ .

4.4 John drives for 50 miles at 75 mph, then drives 60 miles at 80 miles an hour. What is his average speed in mph for the entire trip, rounded to the nearest whole number?

**Answer:** 78 mph.

4.5 Compute:

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{10 \cdot 11} + \frac{1}{11 \cdot 12}$$

**Answer:**  $11/12$ .

ALT1 Find the intersection point of the lines  $4x + 6y = 7$  and  $y = \frac{1}{2}x - \frac{7}{6}$ .

**Answer:**  $\left(2, -\frac{1}{6}\right)$ .

ALT2 If  $a \neq 1$  and  $a^3 - 1 = 0$ , then find the value of  $a^2 + a + 1$ .

**Answer:** 0.

ALT3 If John walks 3 miles north, 4 miles west, 8 miles south, and finally 8 more miles west, then how far is he from his initial position?

**Answer:** 13.