2000 Hoover High School Mathematics Tournament
Algebra I Examination

1. The sides of a right triangle have lengths $x+5$, $x-3$, and $x+1$. Find x.

a. 2 b. 4 c. 8 d. 16 e. NOTA

2. Simplify the following expression:

$$\frac{(a+1)^3(a-1)^2}{a^4-2a^2+1}$$

a. $a+1$ b. $a-1$ c. a^2-1 d. $1-a$ e. NOTA

3. Natalie and Asya can finish a job in 1.5 hours. Megan and Asya can finish in 2 hours, and Natalie and Megan can finish in 1 hour. How long would it take them to finish if all three worked on the job?

a. 3/2 hour b. 2/3 hour c. 12/13 hour d. 13/12 hour e. NOTA

4. How many solutions does the following equation have?

$$6x^3-7x^2-45x-14=0$$

a. 0 b. 1 c. 2 d. 3 e. NOTA

5. Solve for z:

$$\sqrt{z+5}=z-7$$

a. 11, 4 b. 4 c. 11 d. $\varnothing$ e. NOTA

6. Compute the value of the following expression when x is 1089:

$$\left(\sqrt[4]{x}+\frac{1}{\sqrt[4]{x}}\right)^2-\left(\sqrt[4]{x}-\frac{1}{\sqrt[4]{x}}\right)^2$$

a. 132 b. 33 c. 4 d. $4\sqrt{33}$ e. NOTA

7. Find the area bounded by the x-axis, the y-axis, and the line $5x+12y+60=0$.

a. 60 b. 30 c. 15 d. 15/2 e. NOTA

8. If the sum of the squares of the side-lengths of a right triangle is 162, then what is the length of the hypotenuse?

a. $9\sqrt{2}$ b. $8\sqrt{2}$ c. 9 d. 8 e. NOTA

9. If the area of a square of side s is equal to the area of a circle of radius r, then what is the ratio $\frac{s}{r}$?

a. $\frac{1}{\pi}$ b. π c. $\frac{1}{\sqrt{\pi}}$ d. $\sqrt{\pi}$ e. NOTA

10. Two points are selected among the points (3, 2), (3, 8), (9, 2), and (9, 8). Find the probability that the distance between them is 6.

a. 2/3 b. 1/3 c. 1/2 d. 5/6 e. NOTA

11. The slope of the line determined by the two points $(2, a)$ and $(a, 2)$ is:

a. 0 b. No Slope c. 1 d. –1 e. NOTA

12. Define the operation $\otimes$ by $a \otimes b = \frac{(a+b)^2}{ab}$. Find the value of $a \otimes b$ if $a = 2b$.

a. 1 b. 2 c. 3 d. 4 e. NOTA

13. Find the area of an isosceles triangle whose sides have lengths 17, 17, and 30.

a. 240 b. 120 c. 60 d. 30 e. NOTA

14. Find an equation for the line whose x-intercept is (4, 0) and whose y-intercept is (0, 3).

a. $3x - 4y = 12$ b. $4x - 3y = 12$ c. $3x + 4y = 12$ d. $4x + 3y = 12$ e. NOTA

15. Simplify: $\sqrt[5]{\frac{2^2 4^3 8^4 16^5}{81^4 27^3 9^2 3^1}}$

a. $\frac{128}{243}$ b. $\frac{128}{729}$ c. $\frac{256}{243}$ d. $\frac{256}{729}$ e. NOTA

16. If in 6 years I will be the square of my current age, then how old am I now?

a. 3 b. 4 c. 5 d. 2 e. NOTA

17. If $f(x) = 2x - 3$, then compute $f(f(f(f(f(f(3))))))$.

a. 0 b. 6 c. –3 d. 3 e. NOTA

18. If $3A + 2B + C = 9$ and $-6A - 4B = 8$, then find C.

a. 5 b. 9 c. 13 d. 17 e. NOTA

19. Simplify completely:

$$\frac{a}{4} \div \left[-\frac{1}{2a} + 2a \right] \times (2a+1)$$

a. $\frac{a}{4a-2}$ b. $\frac{2a}{2a-1}$ c. $\frac{a}{8a-4}$ d. $\frac{2a^2+a}{8a^2-2}$ e. NOTA

20. Find two times the reciprocal of the square of one more than 11.

a. 1/61 b. 2/121 c. 1/72 d. 1/244 e. NOTA

21. In triangle ABC, point D is selected on BC such that $BD = DC$ and point E is selected on AC such that $AE = EC$. If the area of ABC is 16, then what is the area of CDE?

a. 2 b. 4 c. 6 d. 8 e. NOTA

22. If we have 12 numbers whose sum is 144, then at most how many of them can be negative?

a. 0 b. 1 c. 10 d. 11 e. NOTA

23. If $F = ma$ and $F = \frac{mv^2}{r}$, then what is v?

a. ar b. m^2ar c. $\sqrt{ar}$ d. $m\sqrt{ar}$ e. NOTA

24. $1 + 3 + 5 + 7 + \ldots + 99 + 101 = (?)$

a. 5050 b. 5202 c. 2525 d. 2601 e. NOTA

25. Find the largest positive whole number that does not satisfy the inequality.

$$\frac{2450}{x} - 1 \leq x$$

a. 47 b. 48 c. 49 d. 50 e. NOTA

TIEBREAKERS

1. Multiply: $(a-b)(a^7 + a^5b^2 + a^4b^3 + a^3b^4 + a^2b^5 + ab^6 + b^7)$

2. There are many points whose distance from the point (3, 4) is less than $\sqrt{34}$. Find the area of the region composed of all such points.

3. Find the sum of the first 50 positive integers.