2000 Hoover High School Mathematics Tournament
Algebra II Examination

1. The equation $x^4 + Ax^3 + Bx^2 + Cx + D = 0$ with integers A, B, C, D has as two of its roots $-2i+5$ and $3+i$. What is the value of A?

A) 2 B) 16 C) –2 D) –16 E) NOTA

2. If x and y are positive integers such that $x + x + x + x + x + x + x + x = y + y + y + y - y$, find the minimum possible value of x + y.

A) 0 B) 13 C) 8 D) 5 E) NOTA

3. A matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has its determinant a value of 1.5. What is the determinant of the inverse of this matrix?

A) $\frac{2}{3}$ B) -1.5 C) $\frac{3}{4}$ D) 1 E) NOTA

4. If x<0, what is the value of $\sqrt[5]{32x^5}$?

A) $2x$ B) $-2x$ C) $2|x|$ D) $\pm 2x$ E) NOTA

5. Find the sum of the values of x that satisfy the equation $\sqrt{x+4} = x-8$.

A) 5 B) 17 C) 12 D) –3 E) NOTA

6. x varies jointly as z and y and inversely as the square root of w. If $x = 2$ when $z = 7$, $y = 3$, and $w = 9$, what is x when $x = 5$, $y = 14$ and $w = 16$.

A) 2/7 B) 5 C) 20 D) 5/2 E) NOTA

7. Calculate the sum of the series: $\sum_{n=1}^{\infty} \frac{3}{2}\left(\frac{2}{2+\sqrt{3}}\right)^n$

A) $\frac{3}{2}+\sqrt{3}$ B) $\sqrt{3}$ C) $\frac{\sqrt{3}}{2}$ D) $2+\sqrt{3}$ E) NOTA.

7. The equation $x^3 + 4x^2 + 4x + 4 = 0$ has one real root. This root lies between what two consecutive integers?

A) –1 and 0 B) 1 and 2 C) –4 and –3 D)-3 and –2 E) NOTA

8. Solve for x: $6 + \cfrac{3}{6 + \cfrac{3}{6 + \cfrac{3}{6 + \cfrac{3}{6 + \ddots}}}} = x$

A) $6+\sqrt{3}$ B) 6 C) $3+3\sqrt{3}$ D) $3+2\sqrt{3}$ E) NOTA

10. Two tanks pump water out of a conical tower. Pump A can empty the tower in 5 hours. When pump A and B work together the tower can be emptied in 3 hours. How long, in hours, does it take for Pump B to empty the tank?

A) $\frac{47}{6}$ B) $\frac{15}{2}$ C) 8 D) 8.5 E) NOTA

11. Find the 2000^{th} term of an arithmetic sequence if the 589^{th} term is 458 and the 911^{th} is 4000.

A) 15,979 B) 14,971 C) 13, 710 D) 16,432 E) NOTA

12. A rhombus is drawn connecting the endpoints of the major and minor axes of the ellipse $\left(\frac{x}{911}\right)^2 + \left(\frac{y}{2000}\right)^2 = 1$. A point is chosen at random from inside the ellipse. What is the probability that the point is also inside the rhombus?

A) $\frac{1}{2(911)(2000)}$ B) $\frac{1}{\pi(911)(2000)}$ C) $\frac{2(911))}{\pi(1000)}$ D) $\frac{2(1000)}{911\pi}$ E) NOTA

13. Simplify: $\frac{2000^3 + 64}{2000^3 - 6(2000)^2 + 24(2000) - 32}$

A) 2000 B) $\frac{334}{333}$ C) $\frac{2001}{1997}$ D) $\frac{1998}{2003}$ E) NOTA

14. Find the minimum value of the function $f(x)$ if $-6x+4f(x)=2x^2-7$.

A) $\frac{-7}{4}$ B) $\frac{-14}{3}$ C) $\frac{3}{2}$ D) $\frac{-23}{8}$ E) NOTA

15. A die is rolled six times. Find the probability that a three was rolled exactly twice.

A) $\frac{3125}{15552}$ B) $\frac{625}{46656}$ C) $\frac{1}{36}$ D) $\frac{9375}{23328}$ E) NOTA

16. Solve for x: $-1+x=\sum_{k=1}^{2000}\binom{2000}{k}$

A) 2^{2000} B) $1+2^{1999}$ C) $1+2^{2000}$ D) 2^{1999} E) NOTA

17. If $f(x)=\frac{(x+1)(x+3)(x+5)}{(x-2)(x-3)(x-4)}$, find $f^{-1}(-8)$.

A) –2 B) 0 C) 1 D) $\frac{1}{2}$ E) NOTA

18. For how many values of n does ${}_4C_n=(n+1)!$

A) 0 B) 1 C) 2 D) 3 E) NOTA

19. Find the sum of the entries in matrix X: $\begin{bmatrix}2&1&3\\-5&2&-4\end{bmatrix}\begin{bmatrix}-3&1\\0&1\\3&5\end{bmatrix}=X\begin{bmatrix}6&3\\5&2\end{bmatrix}$

A) 27 B) –5 C) $\frac{5}{3}$ D) –81 E) NOTA

20. Simplify: $\frac{1}{-1+i}+\frac{2+3i}{1+i}$

A) $\frac{3}{2}-\frac{3}{2}i$ B) 2 C) $-1-\frac{3}{2}i$ D) 0 E) NOTA

21. Consider the graphs of $y = 2^x$ and $y = x^2$ where $x \geq 0$. These two graphs intersect at two points. If the line that contains these two points is written as $Ax + By = C$ where A, B, C are relatively prime, find $|A + B + C|$.

A) 1 B) 10 C) 0 D) 13 E) NOTA

22. Solve for x: ln(ln(ln(x-3)))=0

A) $3 + e^2$ B) 3 C) $3 + e^e$ D) e^3 E) NOTA

23. Evaluate the determinant: $\begin{vmatrix} 1 & 2 & 0 & -2 \\ -1 & 4 & 7 & -3 \\ 0 & 5 & 8 & 1 \\ 3 & -1 & 4 & 0 \end{vmatrix}$

A) –375 B) –400 C) 25 D) 7 E) NOTA

24. Solve for x: $\log_2 81 \cdot \log_{27} 125 \cdot \log_{625} 32 = \log_7 x$

A) 5 B) 15707 C) 16707 D) 16807 E) NOTA

25. In how many distinguishable ways can the letters in the word BOOKKEEPER be written?

A) 151,200 B) 453,600 C) 3,628,800 D) 1,814,400 E) NOTA

Tie Breakers

1. Solve for x, given that $x > 0$:

$$\frac{x}{1-x} = \frac{1}{x}$$

2. Find the sum: $\sum_{x=1}^{2000} x(i)^x$, (where $i = \sqrt{-1}$).

3. Find the area bounded by the graphs $y = |x|$ and $y = 4$