

**2000 Hoover High School Mathematics Tournament  
Comprehensive Ciphering**

1.1 If  $\vec{u} = \langle 1, 2, 3 \rangle$  and  $\vec{v} = \langle 3, 2, 1 \rangle$ , then compute the value of  $\vec{u} \cdot (\vec{v} \times \vec{u})$ .

**Answer:** 0.

1.2 Find the remainder when

$$1!+2!+3!+4!+5!+6!+7!+8!+9!+10!$$

is divided by 15.

**Answer:** 3.

1.3 A triangle with side lengths of 41, 9, and 40 is inscribed in a circle. Find the circumference of the circle.

**Answer:**  $41\pi$ .

1.4 How many ordered pairs of positive integers  $(x, y)$  exist such that  $6x + 18y$  divides 92?

**Answer:** 0.

1.5 What is the distance between the points  $(11117, -42, 17.9)$  and  $(11114, -40, 23.9)$ ?

**Answer:** 7.

2.1 Consider the sequence of integers  $a_0, a_1, a_2, \dots$  defined such that  $a_0 = 1$  and

$$a_n = a_{n-1} + 1 - 2n. \text{ Find } a_{100}.$$

**Answer:** -9999.

2.2 Find the volume of the solid generated by rotating an equilateral triangle  $360^\circ$  about one of its altitudes if it has a side of length 12.

**Answer:**  $72\pi\sqrt{3}$ .

2.3 If  $\theta$  is a first quadrant angle such that  $3\cos\theta + 2\sin\theta = 1$ , then compute  $\sin\theta$ .

**Answer:**  $\frac{2 + 6\sqrt{3}}{13}$ .

2.4 Suppose A and B are dependent events such that the probability of A is 0.3, the probability of B is 0.4, and the probability of both A and B is 0.2. Find the probability that either A or B occurs.

**Answer:** 0.5.

2.5 Find the sum of the first 50 positive multiples of 3.

**Answer:** 3825.

3.1 Two numbers are selected uniformly and at random on the interval  $[-1, 1]$ . Find the probability that the sum of their squares is less than 1.

**Answer:**  $\frac{\pi}{4}$ .

3.2 Marge can wash the dishes in 1 hour by herself, but she can finish in 45 minutes with Homer's help. How fast can Homer wash the dishes alone?

**Answer:** 3 hours.

3.3  $\log 200^{1000} - 1000 \log 2 = (?)$

**Answer:** 2000.

3.4 Compute the value of  $x^5$  if  $x + x^2 + x^3 + x^4 = -1$ .

**Answer:** 1.

3.5 Compute the surface area of a regular octahedron with edge length 8.

**Answer:**  $128\sqrt{3}$ .

4.1 An isosceles trapezoid with perimeter 36 has area 72. Find the radius of a circle inscribed in the trapezoid.

**Answer:** 4.

4.2 If

$$\frac{1}{2i} + \frac{1}{4i^2} + \frac{1}{8i^3} + \frac{1}{16i^4} + \cdots = a + bi,$$

then find  $a + b$ .

**Answer:**  $-3/5$ .

4.3 Evaluate:

$$\lim_{\theta \rightarrow 0} \frac{\cos 4\theta - 1}{\theta^2}$$

**Answer:** -8.

4.4 Compute  $\binom{100}{50} - \binom{99}{50} - \binom{99}{49}$ .

**Answer:** 0.

4.5 Suppose the graphs of  $y = x^2 - 4x + 7$  and  $y = 5 - x$  intersect at the points  $(x_1, y_1)$  and  $(x_2, y_2)$ . Find  $y_1 + y_2$ .

**Answer:** 7.

ALT1 Find the slope of the line tangent to the graph of  $(x - 3)^2 + (y - 4)^2 = 25$  at the point  $(7, 1)$ .

**Answer:**  $4/3$ .

ALT2 Compute the sum of the cubes of the first 10 positive integers.

**Answer:** 3025.

ALT3 Find the arithmetic mean of the harmonic mean and the geometric mean of 2 and 18.

**Answer:** 4.8 or  $\frac{24}{5}$