

**2000 Hoover High School Mathematics Tournament
Geometry Ciphering**

1.1 Find the volume between two concentric spheres of radii 3 and 6.

Answer: 252π .

1.2 Triangles ABC and XYZ are similar with $\frac{XY}{AB} = 3$ and $AB + BC = YZ$. Find $\frac{XY}{BC}$.

Answer: 6.

1.3 How many diagonals does a dodecagon have?

Answer: 54.

1.4 If the point $(3, 17)$ is reflected in the origin and then reflected in the x -axis, then what is the sum of the coordinates of the resulting point?

Answer: 14.

1.5 If Jerry walks 1 mile north to meet Elaine, then 0.6 miles east to meet George, and finally 0.2 miles south to meet Kramer, then how far is Jerry from his original starting point?

Answer: 1 mile.

2.1 Points A, B, C , and D lie on a circle such that chord AC intersects chord BD at E . If $AE > EC$, $AC = 8$, $BE = 3$, and $ED = 4$, then what is AE ?

Answer: 6.

2.2 If the area of a regular octagon is 2, then what is the length of one of its sides?

Answer: $\sqrt{\sqrt{2}-1}$.

2.3 How many points with whole number coordinates lie in the interior of the graph of $x^2 + (y + 2)^2 = 9$?

Answer: 25.

2.4 Compute the area of a right triangle whose hypotenuse has length 68 and one of whose legs has length 60.

Answer: 960.

2.5 A nonagon has sides 3, 5, 3, 5, 3, 5, 3, 5, and 3. Find the sum of its interior angles.

Answer: 1260° .

3.1 Find the sum of the lengths of the altitudes in a triangle whose sides are 13, 5, and 12.

Answer: $281/13$.

3.2 Compute the area of the triangle determined by the points $(1, 2)$, $(1, 9)$, and $(5, 113)$.

Answer: 14.

3.3 Point P is 26 units away from the center of circle O . If the length of the tangent to O from P is 24, then what is the area of the circle?

Answer: 100π .

3.4 Compute the surface area of a regular tetrahedron if one of its edges has length 12.

Answer: $144\sqrt{3}$.

3.5 A point E is selected inside a rectangle $ABCD$ such that $AE = EC = 6$. If $BE = 7$, then compute ED .

Answer: $\sqrt{23}$.

4.1 Point D is selected on side BC of triangle ABC such that

$$\text{Area}[ABD] = \text{Area}[ADC] = 16.$$

Compute the ratio $\frac{BD}{BC}$.

Answer: $1/2$.

4.2 An isosceles trapezoid of area 60 is circumscribed about a circle of area 25π . What is the perimeter of the trapezoid?

Answer: 24.

4.3 Let the radius of the inscribed circle of a 3-4-5 triangle be r , and the radius of the circumscribed circle be R . Find $R - r$.

Answer: $3/2$.

4.4 A square is arranged in an equilateral triangle such that two of its vertices lie on the base of the triangle, and one of the remaining vertices lies on each of the other two sides of the triangle. If the length of one side of the square is 6, then what is the length of one side of the triangle?

Answer: $6 + 4\sqrt{3}$.

4.5 What length of the line $4x + 6y = 12$ lies in the first quadrant?

Answer: $\sqrt{13}$.

ALT1 Compute the volume of a right circular cone of height 3 and base radius 9.

Answer: 81π .

ALT2 A sphere of radius 4 is inscribed in a cube. Find the volume of the cube.

Answer: 512.

ALT3 Compute the area of the largest circle that could be contained in a square of area 100.

Answer: 25π .