

**2000 Hoover High School Mathematics Tournament
Geometry Examination**

1. If the intersection point of the line determined by the points (3, 4) and (5, 12) and the line determined by the points (4, 3) and (12, 5) is (x, y) , then compute $x + y$.

- a. $10/3$ b. $12/3$ c. $14/3$ d. $16/3$ e. NOTA

2. What is the contrapositive of the following statement:

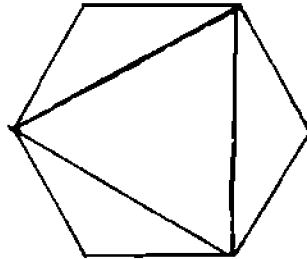
If my dog is Ace, then not all martians are green.

- a. If my dog is not Ace, then all martians are green.
 b. If all martians are green, then my dog is not Ace.
 c. If all martians are not green, then my dog is Ace.
 d. If my dog is Ace, then all martians are green.
 e. NOTA

3. A sphere is both inscribed in a cube and circumscribed about another cube. Find the ratio, large to small, of the edge lengths of the cubes.

- a. $\sqrt{2}$ b. $\frac{2\sqrt{2}}{3}$ c. $\sqrt{3}$ d. $\frac{2\sqrt{3}}{3}$ e. NOTA

4. A triangle is formed by three alternating vertices of a regular hexagon. If this triangle has area $16\sqrt{3}$, then what is the length of one side of the hexagon?



- a. $\frac{8\sqrt{3}}{3}$ b. 8 c. $2\sqrt{3}$ d. 6 e. NOTA

5. Right triangle ABC and square $BCDE$ share side \overline{BC} and \overline{DC} lies on \overline{AC} . If \overline{DE} intersects \overline{AB} at F , $FE = 6$, and $FB = 15.6$, then what is the length of \overline{AD} ?

- a. 16.16 b. 18.16 c. 20.16 d. 22.16 e. NOTA

6. Find an equation for the set of all points equidistant from the point (8, 8) and the point (4, 4).

- a. $x - y = 12$ b. $x + y = 12$ c. $x - y = 6$ d. $x + y = 6$ e. NOTA

7. Compute the length of the longest median in a triangle with side lengths of 8, 10, and 12.

- a. $3\sqrt{10}$ b. $\sqrt{46}$ c. $\sqrt{79}$ d. $\sqrt{106}$ e. NOTA

8. How many triangles exist such that each of its side lengths is a whole number and the sum of its side lengths is 15?

- a. 7 b. 10 c. 13 d. 16 e. NOTA

9. A 6 foot tall man stands 12 feet away from the base of a light post. If the tip of his shadow lies 8 feet in front of him, then how tall is the light post?

- a. 12 b. 13 c. 14 d. 15 e. NOTA

10. Chord AB in circle O is extended to point P outside O . If $AB = BP = 8$, then compute the length of the tangent to O from P .

- a. 8 b. 12 c. $8\sqrt{2}$ d. $12\sqrt{2}$ e. NOTA

11. $ABCDEFGH$ is a cube with edge length 12. Find the volume of tetrahedron $ACFH$.

- a. 576 b. 580 c. 584 d. 588 e. NOTA



12. Six circles of radius 3 are arranged in a circle such that each is tangent to exactly two of the others. Two circles, one of radius r and the other of radius $R > r$, are arranged such that each is tangent to all six of the circles of radius 3. Find the ratio $\frac{R}{r}$.

- a. 2 b. 3 c. 4 d. 5 e. NOTA

13. Consider the line determined by the points $A = (-4, 7)$ and $B = (78, 19)$. If a fly walks along this line $\frac{1}{5}$ of the way to B from A , at what point will he be standing?

- a. $\left(\frac{47}{5}, \frac{62}{5}\right)$ b. $\left(\frac{12}{5}, \frac{82}{5}\right)$ c. $\left(\frac{62}{5}, \frac{47}{5}\right)$ d. $\left(\frac{82}{5}, \frac{12}{5}\right)$ e. NOTA

14. A right triangle with legs a and b and hypotenuse c satisfies $(a + b)^2 = c^2 + 2000$. Find the area of the triangle.

- a. 250 b. 500 c. 1000 d. 2000 e. NOTA

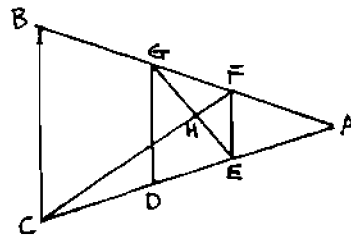
15. While on tour, 'N Sync travelled 1700 miles to Los Angeles from New York. Then they started their 1500 mile trip to Atlanta from Los Angeles. Halfway to Atlanta, they ran out of money and their parents had to fly from New York to pick them up. If Atlanta is due south from New York and due east from Los Angeles, then how far is their parents' flight?

- a. 800 b. 1250 c. 1150 d. 1600 e. NOTA

16. The perimeter of a triangle is 100. Find the numerical value of the area of the triangle divided by the radius of the inscribed circle of the triangle.

- a. 100 b. 200 c. 25 d. 50 e. NOTA

17. Triangle ABC is isosceles with $m\angle B = m\angle C = 72^\circ$. If $m\angle EGB = 136^\circ$, $m\angle EHC = 80^\circ$, and $\overline{FE} \parallel \overline{GD} \parallel \overline{BC}$, then what is $m\angle CFB + m\angle FCE$?



- a. 76° b. 78° c. 80° d. 82° e. NOTA

18. The centers of two circles of radii 4 and 7 are 8 units apart. Find the length of their common external tangent.

- a. $\sqrt{53}$ b. $\sqrt{55}$ c. $\sqrt{57}$ d. $\sqrt{59}$ e. NOTA

19. If the sum of the squares of the lengths of the sides of a right triangle is 32 and its area is 2.25, then what is its perimeter?

- a. 6 b. 7 c. 8 d. 9 e. NOTA

20. If the first-quadrant area bounded by the x -axis, the y -axis, and the line $3x + 4y = 5$ is rotated 360° about the y -axis, then what is the volume of the resulting solid?

- a. $\frac{125}{108}\pi$ b. $\frac{25}{27}\pi$ c. $\frac{108}{125}\pi$ d. $\frac{27}{25}\pi$ e. NOTA

21. Compute the shortest distance from the line $5x + 4y = 40$ to the graph of the equation $x^2 + y^2 - 4x - 6y = -9$.

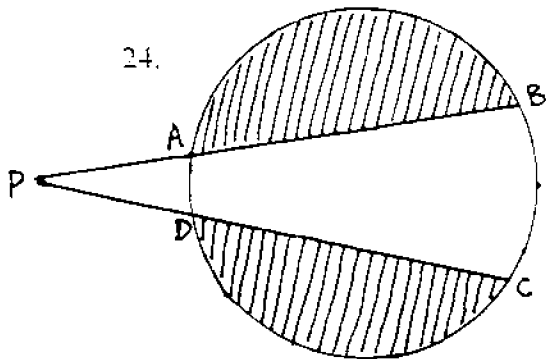
- a. $3/4$ b. $4/5$ c. 1 d. $6/5$ e. NOTA

22. A square is inscribed in a 6-8-10 triangle such that two of its vertices lie on the side of length 10, one of its vertices lie on the side of length 6, and the remaining vertex lies on the side of length 8. Find the length of a side of the square.

- a. $\frac{40}{7}$ b. $\frac{30}{7}$ c. $\frac{160}{37}$ d. $\frac{120}{37}$ e. NOTA

23. A painter, in order to paint a particular pattern he has been working on, must paint a bunch of equilateral triangles of side length 2 in. He divides the work up so that on day one, he only has to paint one triangle, on day two he has to paint two triangles, and so on until day fifteen when he must paint fifteen triangles. How much total paint does he use on all fifteen days (in square inches)?

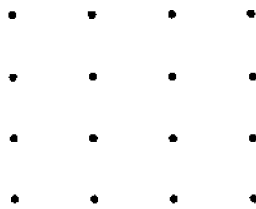
- a. $120\sqrt{3}$ b. $105\sqrt{3}$ c. $30\sqrt{3}$ d. $\frac{105\sqrt{3}}{4}$ e. NOTA



\overline{PB} and \overline{PC} are secants. The area of each of the shaded regions is $12\pi - 9\sqrt{3}$. $m\angle p = 40^\circ$. If the circumference of the circle is 12π , then what is the length of arc AD ?

- a. $\frac{\pi}{3}$ b. $\frac{2\pi}{3}$ c. π d. $\frac{4\pi}{3}$ e. NOTA

25. How many triangles can be formed with vertices in the following grid?



- a. 560 b. 550 c. 540 d. 536 e. NOTA

TIEBREAKERS

1. What type of shape does the following equation describe?

$$2001x^2 - 2000y^2 = 1999$$

2. Find the height of a regular tetrahedron if one of its faces has area $3\sqrt{3}$.
3. There are infinitely many chords of length 2 in a circle of radius 8. Find the area of the region consisting of all such chords.