2001 Hoover High School Mathematics Tournament
Algebra II Examination

1. If $x_1, x_2, ..., x_{2001}$ are all real numbers such that $|x_1| + |x_2| + ... + |x_{2001}| = 0$, find $x_1 + x_2 + ... + x_{2001}$.

A) -2001 B) 0 C) 1000 D) 2,003,001 E) N.O.T.A.

2. Auburn University has an undergraduate enrollment of 22,000. There are a total of 366 possible birthdays for these students (including February 29). Knowing nothing of their birthdays, what is the largest number of these undergraduates that **MUST** share a birthday?

A) 60 B) 61 C) 21,634 D) 21,635 E) N.O.T.A.

3. $\left|35 - 4\sqrt{82}\right|$

A) $4\sqrt{82} - 35$ B) 87 C) $\sqrt{2537}$ D) $35 - 4\sqrt{82}$ E) N.O.T.A.

4. How many times does the graph $y = x^3 + x^2 + x + 1$ cross the x-axis?

A) 0 B) 1 C) 2 D) 3 E) N.O.T.A.

5. What is the constant term in the expansion of $\left(2x^2 - \frac{1}{x}\right)^{15}$?

A) -3003 B) 3003 C) -96096 D) 96096 E) N.O.T.A.

6. What is the largest integer n such that 30^n evenly divides 40! ?

A) 1 B) 7 C) 9 D) 10 E) N.O.T.A.

7. A class has 37 students, all of whom take a test. 13 of the scores had an average of 81on the test while the other 24 scores had an average of 44 on the test. What is the average of all 37 tests?

A) 50 B) 57 C) 62 D) 62.5 E) N.O.T.A.

8. What is the probability that W. C. will be yelled at during the school day if there is a 20% chance that Mr. Sturgeon will yell at him in the morning, a 25% chance that Miss Lutenbacher will yell at him by lunch, and a 50% chance that Miss Baker will yell at him in the afternoon?

A) 70% B) 30% C) 2.5% D) 95% E) N.O.T.A.

9. Simplify the product: $\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right)...\left(1 - \frac{1}{2001}\right)$

A) $\frac{1}{2001}$ B) $\frac{2}{2001}$ C) $\frac{2000}{2001}$ D) $\frac{2}{3}$ E) N.O.T.A.

10. If x = the number of distinct arrangements of the letters in the word **BRICK**, y = the slope of the line $4(x+y)=3x-2(x-y)+17$, and $z=f(3)$ when $f(x)=2x^2-x+5$, find $\frac{xz}{y}$.

A) 3600 B) 1600 C) -9 D) -1600 E) N.O.T.A.

11. If $f(x)=x^2$, find $f(f(f(3)))$.

A) 9 B) 81 C) 6561 D) 43,046,721 E) N.O.T.A.

12. If a and b are real numbers and $\otimes$ is an operations such that $a\otimes b=a+b-\frac{3a}{b}$, find the sum of all c such that $(1\otimes 2)\otimes c=c$.

A) 3 B) $\frac{9}{2}$ C) $\frac{27}{4}$ D) $\frac{13}{2}$ E) N.O.T.A.

13. Solve for x: $2+\cfrac{2}{2+\cfrac{2}{2+\cfrac{2}{x}}}=x$

A) $1+\sqrt{3}$ B) $1-\sqrt{3}$ C) $1\pm\sqrt{3}$ D) $1+\sqrt{2}$ E) N.O.T.A.

14. If $\begin{bmatrix}2 & -4\\ -1 & 3\end{bmatrix}A+\begin{bmatrix}1 & 1\\ 0 & -3\end{bmatrix}=\begin{bmatrix}2 & 0\\ 0 & 4\end{bmatrix}$, find the determinant of matrix A.

A) 7 B) $\frac{7}{2}$ C) $\frac{3}{2}$ D) $\frac{11}{2}$ E) N.O.T.A.

15. If $f(g(x))=x$ and $f(x)=\frac{x}{x+1}$, find $g(x)$.

A) $\frac{x}{x+1}$ B) $\frac{-x}{x+1}$ C) $-\frac{1}{x+1}$ D) $\frac{x}{1-x}$ E) N.O.T.A.

16. Simplify: $\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)^8$

A) 1 B) i C) $\frac{1}{16}+\frac{1}{16}i$ D) -1 E) N.O.T.A.

17. Select an integer from 1 to 500 inclusive. What is the probability that the integer is divisible by 5 or 7, but not both?

A) $\frac{7}{125}$ B) $\frac{171}{500}$ C) $\frac{143}{500}$ D) $\frac{157}{500}$ E) N.O.T.A.

18. What is the units digit in the expression: $1^{2001}+2^{2001}+3^{2001}+\ldots+9^{2001}$?

A) 1 B) 3 C) 5 D) 7 E) N.O.T.A.

19. Find the sum of the solutions of the equation $\sqrt{x+3}+4=\sqrt{8x+1}$.

A) $\frac{22}{49}$ B) 6 C) $\frac{316}{49}$ D) $\frac{272}{49}$ E) N.O.T.A.

20. Given $\log_5 35 = x$, $\log_5 3 = y$, and $\log_3 15 = z$, which of the following expressions is equivalent to $\log_5 7$?

A) $\frac{xy+yz-1}{y}$ B) $\frac{xy-yz-1}{y}$ C) $\frac{xy+yz+1}{y}$ D) $\frac{xy-yz+1}{y}$

E) N.O.T.A.

21. If $x+\frac{1}{x}=7$, find $x^3+\frac{1}{x^3}$.

A) 343 B) 336 C) 322 D) 28 E) N.O.T.A.

22. Let f be a function such that $f(x)=\begin{cases}[x] & , \text{ if } x \text{ is irrational}\\ x-\sqrt{2} & , \text{ if } x \text{ is rational}\end{cases}$

What is the value of $f(f(f(f(f(\pi)))))$? ($[x]$ is the greatest integer function)

A) -1 B) -2 C) $-1-\sqrt{2}$ D) $1-\sqrt{2}$ E) N.O.T.A.

23. Find the determinant of the 4x4 matrix: $\begin{bmatrix} 1 & 3 & 2 & 4 \\ 5 & 2 & 7 & 20 \\ -3 & 1 & 3 & -12 \\ 0 & 1 & 4 & 0 \end{bmatrix}$

A) 620 B) -620 C) -700 D) -740 E) N.O.T.A.

24. If $\frac{6x^3-32x^2+51x-19}{(x+1)(x-3)(x-2)^2}=\frac{A}{x+1}+\frac{B}{x-3}+\frac{C}{x-2}+\frac{D}{(x-2)^2}$ where A, B, C, D are real numbers.

what is $-11A+5B+7C-3D$?

A) -13 B) 25 C) 13 D) -25 E) N.O.T.A.

25. What is the vertex of the parabola $13x^2-36y-182x+565=0$?

A) (1,11) B) (7,-2) C) (-5,50) D) (13,11) E) N.O.T.A.

TIEBREAKERS

1. How many positive integer factors does 19250 have?

2. Find the values of x that make the expression $x^3-x+\frac{1}{x}-\frac{1}{x^3}$ nonnegative

3. Simplify: $25^{\left|\ln(\sqrt{e})-\ln(e^2)\right|}$