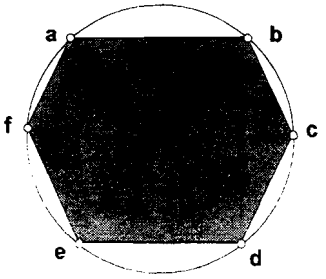
Comprehensive Ciphering - Hoover High School Math Tournament - February 24, 2001

1-1. Find the inverse of $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ Ans: $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

1-2. Evaluate: $\sum_{k=4}^{31} {}_k C_{(k-1)}$ Ans: 490

1-3. $\tan\alpha = \frac{4}{7}$ and $\cos\beta = \frac{3}{5}$; $0 < \alpha < \frac{\pi}{2}$ and $\frac{3\pi}{2} < \beta < 2\pi$. Find $\sin(\alpha - \beta)$. Ans: $\frac{8\sqrt{65}}{65}$

1-4. Figure *abcdef* is a regular hexagon inside a circle of diameter 8. Find the area of the non-shaded region.



Ans: $16\pi - 24\sqrt{3}$

1.5 Evaluate: $\lim_{x\to\infty}\left(\frac{\frac{5x+6}{x^2+5x+6}+\frac{3x+3}{x+2}+\frac{4x-1}{x+3}}{\frac{x+1}{2x+3}+\frac{5x^2-4}{6x^2-x-15}}\right)$ Ans: $\frac{21}{4}$

2-1. Solve $a+b-c-d+e$:

$$\begin{aligned} 2a+b+c+d+e &= -8 \\ a+2b+c+d+e &= 10 \\ a+b+2c+d+e &= 20 \\ a+b+c+2d+e &= 8 \\ a+b+c+d+2e &= 6 \end{aligned}$$

Ans:-26

2.2 Find the determinant of $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -2 \\ 2 & 7 & 3 & 5 & 1 & 5 \\ 0 & 0 & 3 & 6 & 7 & 1 \\ 0 & 0 & 0 & 4 & 5 & 7 \\ 0 & 0 & 0 & 0 & 5 & 1 \\ 3 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. Ans: 2520

2.3 When $e^{\frac{\pi}{2}i}\left(2e^{\frac{\pi}{3}i}\right)$ is simplified and written in $a+bi$ form, find $a+b$. Ans: $1-\sqrt{3}$

2.4 $43_6 + 63_7 + 2_{10} = x_2$. Solve for x. Ans: x = 1001010

2.5 $\dfrac{(x^2-1)(2x^2+10x+12)}{(x^2+2x-3)(x+2)}$ can be expressed in lowest terms as $ax+b$.

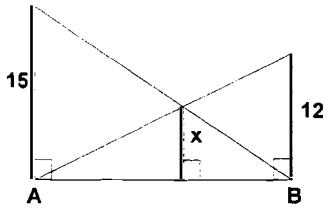
Find $2a+3b$. Ans: 10

3-1. If the probability that Georgia Tech beats the University of Georgia is $\frac{4}{5}$, then what is the probability that Georgia Tech will win exactly 4 out of 5 games? Ans: $\dfrac{256}{625}$

3-2. Laundry soap is 60% water and 40% bubbles. If I add 10mL of bubbles to my 50mL of laundry soap, what percentage of the new solution is water? Ans: 50%

3-3. Solve for x: $7^{19365436} = x \pmod{11}$. Ans: $x=4$

3-4. Find the sum of the heights for the post X when A and B are 5 units apart and when they are 25 units apart.
Ans: 40/3



3-5. Given $x^3+2x^2+3x-7=0$ has roots a,b,c. Solve for $a^2+b^2+c^2$. Ans: -2

4.1 Given that three vertices of parallelogram ABCD are A(5,12), B(0,8) and C(-3,1), find the area of the parallelogram.
Ans: 23

4.2 If the distance between the plane $-2x+3y+z\sqrt{3}=5$ and the point $(a,4,-2\sqrt{3})$ is $\frac{5}{2}$, find the smallest possible value for a. Ans: $\dfrac{-9}{2}$

4.3 At Hoover, 40% of the people have brown hair, 25% have brown eyes, and 15% have both brown hair and brown eyes. A person is selected at random from Hoover. If the person has brown hair, what is the probability that the person also has brown eyes. Ans: 3/8

4.4 Find the sum of $1^2-2^2+3^2-4^2+\ldots-50^2+51^2$. Ans: 1326

4.5 Every ball rebounds to $\frac{1}{2}$ of it preceding maximum height on each successive bounce. A ball is dropped from a height of 40 ft. One minute later, another ball is dropped from half of the previous ball's height. If a new ball is released each minute from half of the height of the preceding ball, how many feet would all of the balls travel?

Ans: 240

E-1 Find the product of the solutions to the equation: $(\log_3 x)^2-6(\log_3 x)+8=0$ Ans: 3^6 or 729

E-2 Evaluate: $\lim_{x\to 0}(1+2x)^{\csc x}$ Ans: e^2

E-3. Evaluate: $\dfrac{4^2x^3y^{\frac{3}{2}}}{3^3x^{-2}y^5z^{-3}}$ Ans: $\dfrac{16x^5z^3}{27y^{\frac{7}{2}}}$