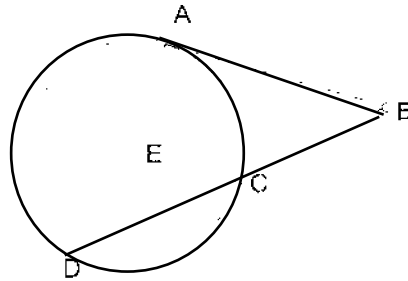


**2001 Hoover High School Mathematics Tournament  
Geometry Examination**

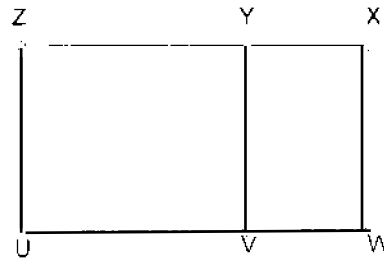
1.  $\overline{AB}$  is tangent to the circle at point  $A$ .  $\overline{BD}$  is a line with segment with  $C$  lying on  $\overline{BD}$ , and points  $A, C,$  and  $D$  lie on the circle. If the length of  $\overline{AB}$  is 6 and the length of  $\overline{BC}$  is 4, what is the length of  $\overline{CD}$ ?

- A) 24  
B) 9  
C) 7  
D) 5  
E) N.O.T.A



2.  $ZUWX$  is a rectangle with  $\overline{UV} = 1$ .  $UVYZ$  is a square, and rectangle  $XYVW$  is similar to rectangle  $ZUWX$ . Find the area of rectangle  $ZUWX$ .

- A)  $\frac{3 - \sqrt{5}}{2}$   
B)  $-2 + \sqrt{5}$   
C)  $\frac{-1 + \sqrt{5}}{2}$   
D)  $-4 + 2\sqrt{5}$   
E) N.O.T.A



3. In the plane, how many lines of symmetry does a regular octagon have?

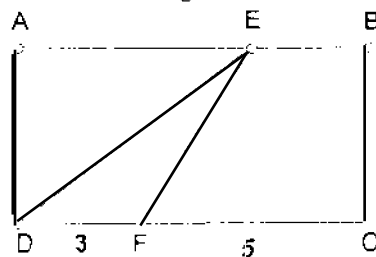
- A) 4      B) 8      C) 16      D) 20      E) N.O.T.A.

4. Four concentric circles are drawn with radii of length 1,3,5,7. Find the sum of the areas between the smallest and second smallest circles and between the second largest and largest circles.

- A)  $9 + 14\pi$     B)  $32\pi$     C)  $54\pi$     D)  $84\pi$     E) N.O.T.A.

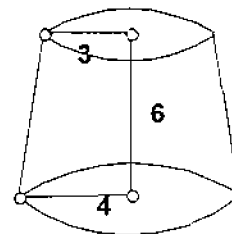
5. Find the ratio of the area of triangle  $DEF$  to the area of rectangle  $ABCD$ .

- A)  $5/3$   
B)  $5/8$   
C)  $3/8$   
D)  $3/16$   
E) N.O.T.A



6. What is the converse of the statement "If I score the highest in the class on any test, then I am in the club."?
- A) If I am not in the club, then I did not score the highest in the class on every test.  
 B) I did not score the highest in the class on every test or I am in the club.  
 C) If I am in the club, then I score the highest in the class on any test.  
 D) If I did not score the highest in the class on every test, then I am not in the club.  
 E) None of the above
7. What is the slope of the line tangent to the graph  $x^2 + y^2 = 169$  at the point  $(-12, -5)$ ?
- A)  $-\frac{12}{5}$       B)  $-\frac{5}{12}$       C)  $\frac{5}{12}$       D)  $\frac{12}{5}$       E) N.O.T.A.
8. Triangle  $ABC$  is inscribed in a circle.  $\angle ABC = 90^\circ$ . The length of  $AB$  is 18, and the length of  $BC$  is 80. What is the area of the circle?
- A)  $169\pi$       B)  $1691\pi$       C)  $6561\pi$       D)  $6724\pi$       E) N.O.T.A.
9. If a convex  $n$ -gon has a total of 230 diagonals, what is the value of  $n$ ?
- A) 13      B) 16      C) 20      D) 23      E) N.O.T.A.
10. A triangle has side lengths 9, 10 and 11. What is the length of the longest altitude of this triangle?
- A)  $\frac{10\sqrt{2}}{3}$       B)  $10\sqrt{2}$       C)  $\frac{20\sqrt{2}}{3}$       D)  $30\sqrt{2}$       E) N.O.T.A.
11. An equilateral triangle of side length 3 is inscribed inside a circle. What is the area of this circle?
- A)  $3\pi$       B)  $\frac{15\pi}{4}$       C)  $9\pi$       D)  $12\pi$       E) N.O.T.A.
12. Quadrilateral  $ABCD$  is inscribed in a circle.  $AB = 6$ ,  $BC = 7$ ,  $CD = 4$ ,  $DA = 8$  and  $AC = 8$ . Find  $BD$ .
- A) 8      B)  $\frac{35}{4}$       C)  $\frac{37}{4}$       D) 10      E) N.O.T.A.
13. A sphere is to be covered with the same amount of liquid polymer used to cover the area of a circle with diameter 12 cm long. Find the radius of the sphere.
- A) 3 cm      B) 6 cm      C) 9 cm      D) 36 cm      E) N.O.T.A.
14. A cone, as shown, has been sliced parallel to the base of radius 4, so there is a top circle of radius 3. The distance between the top circle and the bottom circle is 6. What is the remaining volume?

- A)  $32\pi$   
 B)  $37\pi$   
 C)  $54\pi$   
 D)  $74\pi$   
 E) N.O.T.A.



15. A sphere has the same surface area as its volume. What is this volume?

- A)  $\frac{\pi}{3}$       B)  $\frac{4\pi}{3}$       C)  $36\pi$       D)  $108\pi$       E) N.O.T.A.

16. Two circles, one of radius 8 and the other of radius 12, are situated so that their centers are 42.5 units apart. What is the length of their common internal tangent?

- A) 15      B)  $\frac{45}{2}$       C)  $\frac{75}{2}$       D)  $\frac{\sqrt{7161}}{2}$       E) N.O.T.A.

17. Find the area inside the graph  $|x - 2| + |y - 4| = 2$ , but outside the graph  $(x - 2)^2 + (y - 4)^2 = 2$ .

- A)  $4 - 2\pi$       B)  $8 - 2\pi$       C)  $16 - 2\pi$       D)  $16 - 4\pi$       E) N.O.T.A.

18. A polyhedron has 8 faces and 17 edges. How many vertices does it have?

- A) 7      B) 11      C) 23      D) 27      E) N.O.T.A.

19. A quadrilateral has sides length 24, 25, 26 and 27, and area  $180x$ , where  $x$  is a real number. Find  $x^2$ .

- A) 9      B) 13      C) 39      D) 169      E) N.O.T.A.

20. The length of a rectangle is decreased by 25% and its width is increased by 25%. The area of the new rectangle is 30 units less than the original rectangles' area. Find the area of the original rectangle.

- A) 120      B) 420      C) 540      D) 720      E) N.O.T.A.

21. A space diagonal of a cube has length  $\sqrt{7}$ . What is the surface area of this cube?

- A) 14      B)  $\frac{21}{2}$       C)  $\frac{28}{3}$       D) 7      E) N.O.T.A.

22. A rectangular solid has faces with areas of 15, 21, and 35. What is the volume of the solid?

- A) 11025      B) 210      C) 115      D) 105      E) N.O.T.A.

23. In the coordinate plane, points  $A(-2, -1)$ ,  $B(0,5)$  and  $C(4,3)$  are vertices of parallelogram  $ABCD$ . What are the coordinates of vertex  $D$ ?

- A)  $(-2, 3)$       B)  $(6, -1)$       C)  $(0, -5)$       D)  $(2, 5)$       E) N.O.T.A.

24. A cone has a diameter of 14 inches and a height of 24 inches. A ball of radius 3.5 inches is dropped into the cone. When the ball comes to rest, the bottom of the ball is how many inches above the vertex of the cone?

- A) 12.5      B) 12      C) 9      D) 8.5      E) N.O.T.A.

25. If two angles of a triangle measure  $(90 + x)^\circ$  and  $(90 + y)^\circ$ , where  $x \leq 0$  and  $y \leq 0$ , which of the following must be true?
- A)  $x \geq y$
  - B)  $x = y$
  - C)  $-x \geq y$
  - D)  $-x \leq y$
  - E) N.O.T.A

### **TIE-BREAKERS**

1. An equiangular triangle and a square have the same perimeters. Find the ratio of the area of the triangle to the area of the square.
2. Find the area of a regular hexagon of side length 1.
3. An ant is walking on the coordinate plane. He starts on the point  $(0,0)$  and he may only move to a point if both of its coordinates are integers. Furthermore, the ant may only move up or to the right at any time. How many pathways are possible if the ant wants to reach the point  $(6,5)$ ?