

1996 Hoover High School Math Tournament
Algebra II Written Test
February 24, 1996

1. Solve if $x = 1309$, $y = \frac{2}{17}$, and $z = 17$: $\frac{2x^2yz}{3y^2} \cdot \frac{2x^0z^2}{y^3x^2} \cdot \frac{3xy}{4z^2} \cdot \frac{4x^3y^7z}{x^4z^2}$
- A. 34 B. 1309 C. 154 D. 2 E. none of these
2. Determine the value(s) of k for which $x^2 + 4(k+1)x + 4k^2 = 0$ will have imaginary (conjugate) roots.
- A. $k < -\frac{1}{2}$ B. $-2 < k < 0$ C. $k=0$ D. $k < 0$ E. none of these
3. The illuminating power of a light bulb is equal to the amount of light received by an object times the square of the distance from the bulb to that object. Two light bulbs with illuminating powers of 25 and 100 are 32 feet apart. A card is placed between the two bulbs so as to receive the same amount of light from each. How far from the card is the bulb with illuminating power of 100?
- A. 8 feet B. $10\frac{2}{3}$ feet C. 10 feet D. $21\frac{1}{3}$ feet E. none of these
4. Simplify: $\sqrt[3]{\sqrt{27}} \cdot \sqrt{\sqrt{27}}$
- A. $\sqrt[4]{27}$ B. 3 C. $3\sqrt{3}$ D. 9 E. none of these
5. If $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} X = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$, find the sum of the entries of matrix X.
- A. 16 B. 1 C. 2 D. 0 E. none of these

6. $\left(\frac{1}{2} + \frac{1}{3}\right) + \left(\frac{1}{4} + \frac{1}{9}\right) + \left(\frac{1}{8} + \frac{1}{27}\right) + \dots + \left(\frac{1}{2^n} + \frac{1}{3^n}\right) + \dots = ?$
- A. 1.5 B. 1 C. $\frac{2}{3}$ D. 2 E. none of these
7. Find the sum of all k such that $(x+k)$ is a factor of $kx + 2x^2 + k + kx^2 + kx^3 + 2x$.
- A. 0 B. 1 C. 2 D. -1 E. none of these
8. $1996.6875_{10} = ?_2$
- A. 11111001100.1101011011011 B. 11111001100.101
- C. 11111001100.01101001 D. 11111001100.1011
- E. none of these
9. How many odd numbers between 450 and 700 can be formed using only the digits 3,4,5,6,7, and 8?
- A. 96 B. 90 C. 72 D. 48 E. none of these
10. Evaluate: $\frac{-33 + 56i}{5 - 12i}$
- A. 2-i B. 5 C. 3 D. 3+i E. none of these
11. If $f(x) = \frac{3x-4}{2x-3}$, find $[f^{-1}(x)][f(x)]^{-1}$.
- A. $\left(\frac{2x-3}{3x-4}\right)^2$ B. $\left(\frac{3x-4}{2x-3}\right)^2$ C. 1 D. $\left(\frac{2x-3}{3x-4}\right)$ E. none of these
12. What is the shape of the figure given by $4x^2 + 9y^2 - 24x + 36y + 72 = 0$?
- A. parabola B. ellipse C. hyperbola D. circle E. none of these

13. Determine the value(s) of k so that one root of $kx^2 - 3x + 10 = 0$ is twice the other.

- A. $\frac{1}{5}$ B. $\frac{1}{5}, -\frac{1}{5}$ C. 2 D. $-\frac{1}{5}$ E. none of these

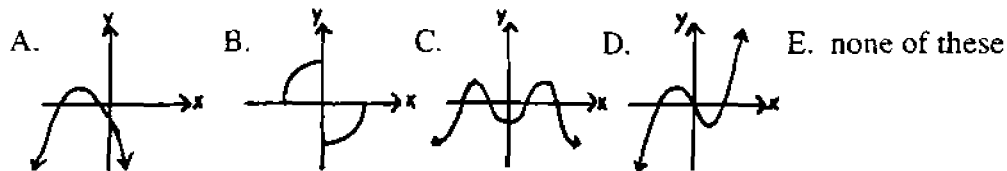
14. Consider the number: $(1996)(1995)(1994)(1993)\dots(4)(3)(2)(1)$. In how many zeros does it end?

- A. 597 B. 454 C. 600 D. 516 E. none of these

15. Solve for n : $\frac{(n+1)!}{(n^4 - n^3)(n-2)!} = \frac{21}{400}$

- A. $-\frac{20}{21}$ B. 5 C. 16 D. 20 E. none of these

16. Which of the following graphs has even symmetry?



17. Solve over \mathbb{R} : $\sqrt{y+6} - \sqrt{4-y} = 2$.

- A. -5 B. -5, 3 C. 3 D. -3, 5 E. none of these

18. Solve the system of equations for $x + y + z$:
 $2x + 3y + z = 4$
 $3z - y - 5x = -1$
 $x + y + 2z = 9$

- A. 1 B. 7 C. 4 D. 5 E. none of these

19. If the difference $\sqrt[3]{36} - .36$ is written as $\frac{m}{n}$, where m and n are relatively prime, find $m-n$.

- A. -274 B. 0 C. 274 D. -63 E. none of these

20. Find the constant term in the expansion of $\left(\frac{2x}{y^2} + \frac{y}{\sqrt{x}}\right)^9$.

- A. 84 B. 672 C. 168 D. 8 E. none of these

21. How many negative roots are in the equation $x^6 - x^4 + x - 13 = x^5 - 3x^3 + 17x^2$?

- A. 0 B. 1 C. 2 D. 3 E. none of these

22. Five cards are to be chosen at random from a standard 52-card deck. What is the probability of getting a flush? (A flush is a five card hand where all five cards are of the same suit.)

- A. $\frac{33}{16660}$ B. $\frac{33}{66640}$ C. $\frac{33}{8330}$ D. $\frac{5}{52}$ E. none of these

23. Find the minimum value of the function: $f(x) = -6x^2 + 36x - 43$

- A. 3 B. 11 C. -6 D. -31 E. none of these

24. Evaluate: $\frac{(\log_2 3)(\log_7 25)(\log 4)(\log_{\sqrt{5}} 289)}{(\log 17)(\log_7 9)}$

- A. $\log 4$ B. 8 C. 7 D. $\sqrt{5}$ E. none of these

25. Find the value of the determinant:

$$\begin{vmatrix} 3 & 4 & -2 & 1 \\ 5 & 1 & 0 & -1 \\ 0 & -3 & 2 & 3 \\ 4 & -1 & 0 & -5 \end{vmatrix}$$

A. -46

B. -66

C. 314

D. 147

E. none of these

TIE BREAKERS

TB1 What is the harmonic mean of 2 and 6?

TB2 Simplify:
$$\frac{\sqrt{x^2+x+1}-\sqrt{x^2-x+1}}{\sqrt{x^2+x+1}+\sqrt{x^2-x+1}} + \frac{\sqrt{x^4+x^2+1}-1}{x}$$

TB3 If $f(n) = \frac{1}{\sqrt{5}} \left[\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right]$, find $\sum_{n=1}^9 f(n)$.