

**1996 Hoover High School Math Tournament
Comprehensive Ciphering**

1.1 If a nickel is a right circular cylinder 2 cm in diameter, .5 cm in height, and 7 g in mass, find the density of the nickel in g/cm^3 .

ANS.: $\frac{14}{\pi}$

1.2 At what time between 2 p.m. and 3 p.m. do the hands of a clock come together?

ANS.: 2:10 $\frac{10}{11}$

1.3 Find the value of $\tan^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{1}{9}\right)$.

ANS.: 45° or $\frac{\pi}{4}$

1.4 If $x + \frac{1}{x} = \sqrt{3}$, find $x^5 + \frac{1}{x} + \frac{1}{x^5} + x$.

ANS.: 0

1.5 How many permutations are there of the word **BUCCANEERS** ?

ANS.: 907200

2.1 If $f(x) = \sqrt[3]{8 - 12x + 6x^2 - x^3}$, find $f(-2)$.

ANS.: 4

2.2 Evaluate $\left| \frac{7 - 24i}{3 + 4i} \right|$ where $i = \sqrt{-1}$.

ANS.: 5

2.3 Solve for x : $\frac{x^2 - x - 6}{x^2 - x - 2} \geq 0$

ANS.: $(-\infty, -2] \cup (-1, 2) \cup [3, \infty)$ or $x \leq -2, -1 < x < 2, \text{ or } x \geq 3$

2.4 The shortest distance from a point to a circle is 4 and the longest distance is 9. Find the length of a tangent from the point to the circle.

ANS.: 6

2.5 The probability of rolling n one's on n regular six-sided dice is $\frac{1}{45}$ the probability of rolling $n-2$ five's and two ones with the same dice. Find n .

ANS.: 10

3.1 Solve for a : $\ln a = \ln(a+2) + \ln(a-1)$

ANS.: $\sqrt{2}$

3.2 If $r, s,$ and t are the roots of $x^3 - 5x^2 + 7x - 9$, find $\frac{1}{r} + \frac{1}{s} + \frac{1}{t}$.

ANS.: $\frac{7}{9}$

3.3 Simplify in terms of either sin or cos: $\cot 10 + \tan 5$

ANS.: $1/\sin 10$

3.4 Find the maximum value of $\frac{10}{2x^2 + 6x + 9}$

ANS.: $20/9$

3.5 A right triangle (with integral sides) has two sides of length 33 and 545. Find its perimeter.

ANS.: 1122

4.1 Find the sixteenth number of the nineteenth row of Pascal's Triangle.

ANS.: 816

4.2 Find the focal width of $y^2 + 10y - 9x + 16 = 0$

ANS.: 9

4.3 If $\vec{A} = 2i + 3j - 4k$ and $\vec{B} = 16i - 6j + 9k$, find $(\vec{A} \times \vec{B}) \cdot \vec{A}$

ANS.: 0

4.4 Simplify for $\pi < x < 2\pi$, $x \neq \frac{3\pi}{2}$: $\sqrt{\frac{2 \sin x - \cos x \sin 2x}{\sin 2x \sec x}}$

ANS.: $-\sin x$

4.5 Jim wishes to create a board game that needs to have all of the integral dollars from \$1 to \$63. He can only have one bill of each denomination and each bill can represent any amount of money (in integral dollars). What is the least number of bills that he needs?

ANS.: 6

ALT: Solve for y: $\log(2y-3) + \log(6x+2) = \log(4x - \frac{2}{3}) + \log(3y)$

Ans.: $y = 3x+1$

ALT: If $F(x,y,z) = \frac{5(x^2 + y^2 + z^2) - xy + z - 2}{xyz}$, find $F(12,16,21)$

ANS.: 1