1996 Hoover High School Math Tournament
Comprehensive Written Test
February 24, 1996

1. Let $G_n = G_{n-1} + G_{n-2} + G_{n-3}$ and $G_1 = G_2 = G_3 = 1$. Find the value of G_6.

   A. 5 B. 9 C. 17 D. 29 E. none of these

2. Find the units digit of $1 + 2 + 3 + 4 + ... + 1995 + 1996$.

   A. 4 B. 5 C. 6 D. 7 E. none of these

3. If $m + n + p = 6$ and $mnp = 3$, find $\frac{1}{mp} + \frac{1}{np} + \frac{1}{nm}$.

   A. 2 B. 6 C. 3 D. 19 E. none of these

4. Find the term without the u in $\left(u^2 + \frac{1}{u}\right)^6$

   A. 15 B. 20 C. 6 D. 21 E. none of these

5. Find the smallest integer, greater than 1, that leaves a remainder of 1 when divided by 2, 3, 4, 5, 6, ..., 10.

   A. 1261 B. 5041 C. 2521 D. 841 E. none of these

6. Determine the value of $\log\frac{1}{2} + \log\frac{2}{3} + \log\frac{3}{4} + ... + \log\frac{99}{100}$.

   A. 2 B. -2 C. 1/2 D. -1/2 E. none of these

7. Find the sum of the elements of the fifth row of Pascal's triangle.

   A. 31 B. 64 C. 128 D. 16 E. none of these

8. Quadrilateral ABCD inscribed in a circle with $m\angle ACB = 40°$, $m\angle BDC = 20°$, and $m\angle ACD = 20°$. Find $m\angle DBC$.

   A. 60° B. 40° C. 100° D. 50° E. none of these

9. George buys 5 notebooks and 1 pencil for $2.80, and Craig buys 5 pencils and 1 notebook for $5.60. How much does it cost to buy 2 notebooks and 3 pencils?

   A. $1.85 B. $3.85 C. $3.70 D. $3.50 E. none of these

10. Simplify $\sqrt{6+\sqrt{11}} - \sqrt{6-\sqrt{11}}$

    A. 2 B. -2 C. $\sqrt{2}$ D. $-\sqrt{2}$ E. none of these

11. The sum of the first 3n positive integers is 68 more than the sum of the first n positive integers. What is n?

    A. 3 B. 4 C. 5 D. 6 E. none of these

12. Find all x such that $x+x^3+x^5+x^7+\ldots=2$.

A. $\frac{-1+\sqrt{17}}{4}$ B. $\frac{-1\pm\sqrt{17}}{4}$ C. $\frac{1-\sqrt{17}}{4}$ D. $\frac{1\pm\sqrt{17}}{4}$ E. none of these

13. If P is the period of, A the amplitude of, and S the phase shift of $7\sin(4x+\pi)+3$, find the value of $\frac{3P}{S}+2A$.

A. 11 B. 14 C. 2 D. 8 E. none of these

14. Solve for ϕ: $\begin{vmatrix} \phi & 2 & 3 \\ 2 & \phi & -3 \\ 3 & -1 & 2 \end{vmatrix}=0$

A. 8, -2 B. 6, -4 C. 10, 8 D. -4, -2 E. none of these

15. Given that $\log 2 \approx .3010$, how many digits are there in 2^{100} ?

A. 31 B. 30 C. 29 D. 28 E. none of these

16. How many integral solutions are there to $\left(x^2-5x+5\right)^{x^2+2x+1}=1$?

A. 3 B. 4 C. 5 D. 6 E. none of these

17. Three four-sided dice, (sides numbered one, two, three, four), are rolled. What is the probability of rolling a sum of 10 or higher?

A. 3/64 B. 9/64 C. 5/32 D. 15/64 E. none of these

18. Find the number of integers x such that $\binom{20}{3x}=\binom{20}{x^2-2x}$. $\left[\text{Note that } \binom{m}{n}=\frac{m!}{n!(m-n)!}\right]$

A. 6 B. 4 C. 3 D. 1 E. none of these

19. If x<y<z , xyz = 8, x + y + z = 1, and xy + yz + zx = -10, find the value of $x^3+y^3+z^3$.

A. 63 B. -1 C. 7 D. -3 E. none of these

20. How many solutions $0 \le x \le 2\pi$ are there to $\cos(x)+\cos(2x)+\cos(4x)=0$?

A. 10 B. 8 C. 6 D. 4 E. none of these

21. How many terms are there in the simplified expansion of $(C+A+P)^{15}$?

A. 680 B. 455 C. 120 D. 136 E. none of these

22. A circle contains the points (14, 22.5), (11, 19), and (28,45). Find the length of the tangent drawn to the circle from the origin.

A. 17 B. 28 C. 19 D. 25 E. none of these

23. A hyperbola and a circle share a center and intersect in four points. The circle also passes through the hyperbola's foci F_1 *and* F_2. Let P be any one of the intersection points. If ΔPF_1F_2 has area 30 and the transverse axis of the hyperbola has length 11, find the length of F_1F_2.

A. $\sqrt{271}$ B. $\sqrt{241}$ C. $\sqrt{211}$ D. $\sqrt{181}$ E. none of these

24. If (x_0, y_0, z_0) is the point on the plane 2x + y - z = 5 that is closest to the origin, find the value of $x_0y_0z_0$.

A. $-\frac{125}{108}$ B. $-\frac{168}{289}$ C. $-\frac{225}{289}$ D. $\frac{115}{216}$ E. none of these

25. Let $\left(x+2x^2+3x^3+4x^4+...+10x^{10}\right)^2 = a_1x+a_2x^2+a_3x^3+a_4x^4+...+a_{20}x^{20}$ Find the value of $a_{11}+a_{12}+a_{13}+a_{14}+...+a_{20}$.

A. 495 B. 2380 C. 2530 D. 3876 E. none of these

TIE BREAKERS

TB1. A matrix, when multiplied by itself, yields a matrix with determinant 16. Find the minimum value of the determinant of the original matrix.

TB2. Find $.\overline{623}_7$ as a base ten fraction.

TB3. If $c_n = n + \cfrac{1}{2n + \cfrac{1}{2n + \ddots}}$, find the value of $\sum_{i=1}^{7} c_i^2$.