

1996 Hoover High School Math Tournament
 Geometry Written Test
 February 24, 1996

1. The measures of the interior angles of a triangle are given by x , $2x$, and $4x-30$. Find the sum of the smallest and largest angles.

A. 60° B. 90° C. 30° D. 120° E. none of these
2. $\triangle ABC$ is similar to $\triangle DEF$ with $AB=2$, $DE=6$, and $EF=8$. Find BC .

A. $\frac{8}{3}$ B. 24 C. 1 D. $\frac{4}{9}$ E. none of these
3. A regular n -gon has 170 diagonals. What is the measure of an exterior angle?

A. 20° B. 18° C. 9° D. 10° E. none of these
4. A triangle has two sides of length 6 and 9. How many integral values may the third side attain?

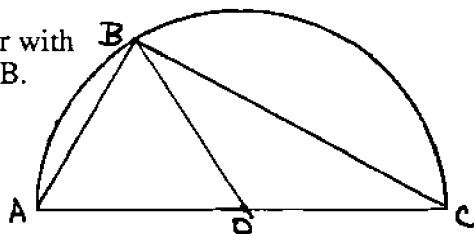
A. 13 B. 12 C. 11 D. 10 E. none of these
5. Which of the following represents the equation of the line (in intercept form) that contains the points $(5,0)$ and $(0,12)$?

A. $y = \frac{-12}{5}x + 12$ B. $\frac{x}{5} + \frac{y}{12} = 1$ C. $5x + 12y = 60$ D. $5x + 12y - 60 = 0$

E. none of these

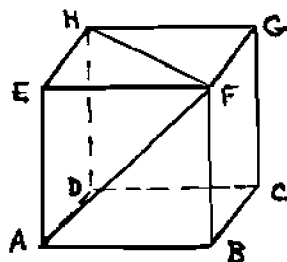
6. Find the area of the triangle formed by the points $(7, 8)$, $(12, 5)$, and $(-4, 3)$.

A. 97 B. 39 C. 58 D. 29 E. none of these
7. In circle O , \overline{AC} is a diameter with $AB=9$ and $BC=12$. Find OB .



- A. $\frac{15}{2}$ B. 15 C. 21 D. $\frac{21}{2}$ E. none of these

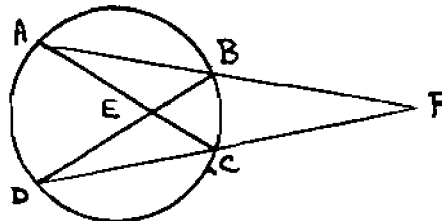
8. ABCDHGFE is a cube. Find the measure of $\angle AFH$.



- A. 90° B. 60° C. 30° D. 15° E. none of these

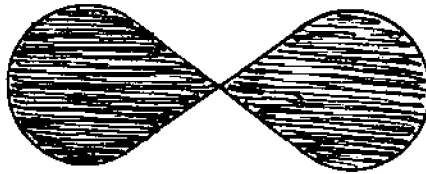
9. A sphere of volume 2 is inscribed in a cylinder. What is the volume of the cylinder?
- A. 4 B. $\frac{4\pi}{3}$ C. 3 D. $\frac{7}{2}$ E. none of these
10. Find the area of the annulus formed by the inscribed and circumscribed circles of a regular hexagon of side length 6.
- A. 36π B. 16π C. 20π D. 9π E. none of these
11. Mark walks 2 miles east, 6 miles north, 6 miles east, and then straight back to his starting point. How far has he walked?
- A. 10 B. 24 C. 14 D. 20 E. none of these
12. From a point within an equilateral triangle, perpendiculars of length 3, 6, and 9 are dropped to the sides. Find the area of the triangle.
- A. $54\sqrt{3}$ B. $216\sqrt{3}$ C. $108\sqrt{3}$ D. $96\sqrt{3}$ E. none of these
13. What is the distance between the incenter and circumcenter of a triangle with sides of length 12, 16, and 20?
- A. 4 B. 5 C. $2\sqrt{5}$ D. $2\sqrt{7}$ E. none of these
14. A right triangle's hypotenuse has projections of length 4 and 9. Find the area of the triangle.
- A. 39 B. 78 C. 36 D. 18 E. none of these

15. \overline{AP} and \overline{DP} are secants of a circle that intersect a circle at B and C respectively. $m\angle APD = 20^\circ$ $m\angle BEC = 40^\circ$
Find $m\angle ABE$.



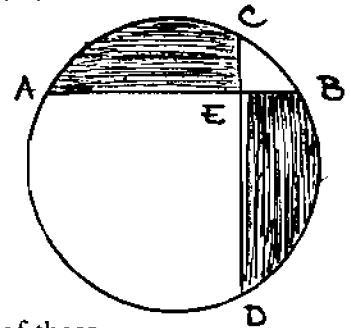
- A. 30° B. 40° C. 60° D. 80° E. none of these
16. A 6 ft. tall man is standing 3 ft. away from a 9 ft. tall light source. How long is his shadow?
- A. 9 ft. B. 8 ft. C. $\frac{9}{2}$ ft. D. 6 ft. E. none of these
17. Find the locus of points, in space, that describes the points that are equidistant from the vertices of a triangle.
- A. a point B. a line C. a plane D. a cube E. none of these
18. The centroid of a triangle is given by the point (3, 5). If two of the vertices are (1, 6) and (9, 4), find the coordinates of the third vertex.
- A. (-11, -8) B. (-8, -11) C. (-1, 5) D. (-4, 4) E. none of these

19. A silo is created by attaching a hemisphere to the top of a right-circular cylinder with an equal radius. If h denotes the height of the silo and r the radius of the cylinder, then $h - \frac{1}{3}r = 5$. The volume of the silo is 80π . Find its surface area (including the bottom).
- A. 40π B. $\frac{152\pi}{3}$ C. $\frac{196\pi}{3}$ D. $\frac{200\pi}{3}$ E. none of these
20. A trapezoid is inscribed and circumscribed about two circles. The bases of the trapezoid are 4 and 8. What is the area of the trapezoid?
- A. 8 B. $12\sqrt{2}$ C. $15\sqrt{2}$ D. 16 E. none of these
21. Find the volume of a regular octahedron with a side of length 6.
- A. $72\sqrt{2}$ B. $78\sqrt{2}$ C. $36\sqrt{2}$ D. $54\sqrt{2}$ E. none of these
22. An ice skater makes a "figure eight" by skating out the region formed by two circles and their internal tangents as shown. If the circles both have radius 4, and the distance between the centers of the circles is 16, find the area that is skated out.



- A. $32\sqrt{3} + \frac{64\pi}{3}$ B. $16\sqrt{3} + \frac{64\pi}{3}$ C. $32\sqrt{3} + \frac{32\pi}{3}$ D. $16 + \frac{32\pi}{3}$ E. none of these
23. Find the radius of the circle circumscribed about a triangle whose vertices are $(5, 3)$, $(10, 7)$, and $(9, -2)$.
- A. $\sqrt{82}$ B. $2\sqrt{41}$ C. $\frac{\sqrt{82}}{2}$ D. $\frac{\sqrt{41}}{2}$ E. none of these
24. In $\triangle ABC$, there is a point P on \overline{BC} such that \overline{AP} bisects $\angle BAC$. If $AB=9$, $AC=6$, and $BP=6$, find AP .
- A. $2\sqrt{6}$ B. $\sqrt{30}$ C. $\sqrt{5}$ D. $\sqrt{10}$ E. none of these

25. In the circle, $\overline{AB} \perp \overline{CD}$ and \overline{AB} intersects \overline{CD} at E . If $AE = DE = 4\sqrt{3}$ and $BE = 2\sqrt{3}$. Find the area of the shaded region.



- A. $\frac{15}{2}\pi - 9$ B. $30\pi - 18$ C. $\frac{15}{2}\pi - 12$ D. $15\pi - 6$ E. none of these

TIE BREAKERS

TB1. What is the perpendicular distance from $28x+45y=106$ to the origin?

TB2. Bob and Jim are running around a 1 mile circular track in the same direction. Bob runs at 8 mph, and Jim runs at 6 mph. If they start from the same point, how long(in hours) will it take for Bob to lap Jim twice?

TB3. A square has the same perimeter as a hexagon. What is the ratio of their areas (smaller to larger)?