1997 Hoover High School
Algebra II Ciphering

1.1 $\sum_{n=0}^{1996}(2n+1) = x^2$. Find x.
answer: 1997.

1.2 Simplify if x is a positive integer greater than 4:

$$\sqrt[x]{x^{\sqrt[x]{x^{\sqrt[x]{x}\cdots}}}}$$

answer: x.

1.3 Find the remainder when $x^{1997} - 1997x + 1997$ is divided by $(x-1)$.
answer: 1.

1.4 If a + b = 6 and ab = 18, find $a^4 + b^4$.
answer: -648.

1.5 Compute:

$$\begin{vmatrix} 4 & 3 & 2 & 1 \\ 0 & 0 & 6 & -1 \\ 1 & 4 & 5 & -3 \\ 0 & 7 & 3 & 4 \end{vmatrix}.$$

answer: -771.

2.1 If the roots of $x^3 - 6x^2 + 11x + k$ are all positive and in arithmetic progression, find k.
answer: -6.

2.2 Find the minimum value of the function $f(x) = 3x^2 + 7x - 11$.
answer: -181/12.

2.3 Given that log2 = 0.30103, find log125.
answer: 2.09691.

2.4 A drawer contains 5 black socks, 4 blue socks, 3 red socks, 2 white socks, and 1 green sock. Five socks are to be chosen at random from the drawer. What is the probability that they are each a different color?
answer: $\frac{40}{1001}$.

2.5 If $\frac{A}{2x-1} + \frac{B}{3x+7} = \frac{2x+16}{6x^2+11x-7}$, find $10A + B$.
answer: 18.

3.1 In slope-intercept form, what is the equation of the asymptote having positive slope

$$144x^2 - 25y^2 + 1440x + 150y - 225 = 0$$

answer: $y = \frac{12}{5}x + 15.$

3.2 If $f(x) = 2^{x+1}$ and $g(x) = -1 + \log_2 x$, find $f(g(f(g(f(g(f(g(f(g(1997)))))))))).$
answer: 1997

3.3 If x is both the first term and the sum of 23 numbers in an arithmetic sequence, while n is the last term of the sequence, find n in terms of x.
answer: $-\frac{21x}{23}.$

3.4 Solve for x: $\left(\left(27^{(1/24)}\right)^6\right)^2 = 3^x.$

answer: 3/2

3.5 If $z = \frac{1}{2} + \frac{\sqrt{3}}{2}i$, find $\overline{(|z|/z)}$ in terms of z.
answer: z.

4.1 Solve over R:

$$\sqrt{x+12} - \sqrt{13-x} = 1$$

answer: 4

4.2 Find the fourth term of the expansion of $(x+3)^{1/2}$.
answer: $\frac{27}{16}x^{-5/2}.$

4.3 Evaluate: $\binom{6}{0} + \binom{6}{1} + \binom{6}{2} + \binom{6}{3} + \binom{6}{4} + \binom{6}{5} + \binom{6}{6}.$

answer: 64.

4.4 Given that:

$$\begin{aligned} 2a + b + c + d + e + f &= 16 \\ a + b + 3c + d + 2e + f &= 12 \\ -a - 2b - 2c + d - 3e - f &= 7 \\ 3a + 5b + 2c + 3d + 3e + 2f &= 14 \\ 2a + 2b + 3c + d + 4e + 4f &= 0 \end{aligned}$$

find the value of $a + b + c + d + e + f$.
answer: 7.

4.5 Find the area of the region bounded by the graphs $y = 0$, $y = x$, and $y = -x + 2$.
answer: 1.

ALT1 Find the two intersection points of $y = 4^x$ and $y = 2^{x+2} - 3$.

answer: $(0,1)$
$(\log_2 3, 9)$.

ALT2 Find the area of the graph of $|x-3| + |y+5| \le 9$.

answer: 162.

ALT3 $1^3 + 2^3 + 3^3 + \ldots + (n-1)^3 + n^3 = (?)$

answer: $\frac{n^2(n+1)^2}{4}$.