

1997 Hoover High School Math Tournament
Algebra II Written Test
February 22, 1997

1. What integer solves this equation?

$$3x^5 - 177x^4 - 7x^3 + 413x^2 - 2x + 118 = 0$$

- A. 69 B. 1997 C. 1 D. 100 E. none of these

2. Find a positive number b such that the sum of the reciprocals of $b + 2$ and $b + 1$ equals the reciprocal of b .

- A. -3 B. 4.1 C. $\sqrt{2}$ D. 1 E. none of these

3. Find the value of x that solves the equation $1 + x + x^2 + x^3 + \dots = \frac{38}{65}$.

- A. $-\frac{21}{34}$ B. $-\frac{27}{38}$ C. $-\frac{13}{19}$ D. $-\frac{17}{24}$ E. none of these

4. Evaluate:

$$\frac{{}_{1997}P_8}{{}_{1997}C_8}$$

- A. 1997 B. 1989! C. 8 D. 1 E. none of these

5. If $f(x) = x^2 + 3x + 2$, then find the value of: $\frac{f(x+h) - f(x)}{h}$.

- A. $2x + 3$ B. 0 C. not defined D. $2x + h + 3$ E. none of these

6. In how many ways can you arrange the letters in the word DALESANDRO?

- A. 3,628,800 B. 907,200 C. 1,814,400 D. 1,000,050 E. none of these

7. Which of the following are odd functions?

- I. $f(x) = x + 3$ IV. $f(x) = x^3 - x$
 II. $f(x) = x$ V. $f(x) = 2x$
 III. $f(x) = 3$ VI. $f(x) = x^4 + x^3 + x^2 + x + 1$

- A. II, IV, V B. I, III, VI C. IV, VI D. II, III, IV E. none of these

8. Simplify:

$$\sqrt[4]{\sqrt{x}} \cdot \sqrt[3]{\sqrt{x^7}} \cdot \sqrt[5]{\sqrt[3]{x^{13}}}$$

- A. $x^{29/30}$ B. x C. x^2 D. $x^{59/60}$ E. none of these

9. If Bighead the Showoff has a $\frac{1}{3}$ probability of making a shot, find the probability that he makes exactly 4 shots out of 5.

- A. $\frac{10}{243}$ B. $\frac{2}{243}$ C. $\frac{1}{243}$ D. $\frac{16}{243}$ E. none of these

10. Simplify: $((1+i)^3 - 2\sqrt{2})((1+i)^3 + 2\sqrt{2})$.

- A. 1 B. $-8i$ C. $2 + 2i$ D. $-8 - 8i$ E. none of these

11. Find the 1997th term of the following series:

$-2, 5, 12, 19, \dots$

- A. 11,742 B. 12,587 C. 13,970 D. 14,710 E. none of these

12. Drake, the mad scientist, has 4000 L of a 21% Rogaine solution. He is going to concentrate the solution by draining some of the solution away and replacing it with 50% solution, yielding him with a 42.025% solution. How much of the 21% solution remained just after draining?

- A. 2900 L B. 2319 L C. 1681 L D. 1100 L E. none of these

13. Katie varies jointly as Young and the square of Morris, and inversely as the square root of Nichols. If Katie = 8 when Young = 4, Morris = 2, and Nichols = 25, then find Nichols when Katie = 17, Young = $\frac{1}{625}$, and Morris = 100.

- A. $\frac{1600}{289}$ B. $\frac{4}{7225}$ C. $\frac{2}{85}$ D. $\frac{40}{17}$ E. none of these

14. $5 + \frac{1}{2 + \frac{1}{5 + \frac{1}{2 + \frac{1}{\dots}}}} = x$. Find x .

- A. $\frac{5 + \sqrt{35}}{2}$ B. $\frac{5 + \sqrt{21}}{2}$ C. $\frac{5 \pm \sqrt{35}}{2}$ D. $\frac{109}{20}$ E. none of these

15. If A and B are both 2×2 matrices where

$$A \cdot B = \begin{bmatrix} -9 & 6 \\ 1 & -2 \end{bmatrix} \text{ and } A^{-1} = \begin{bmatrix} -1/6 & -1/2 \\ -1/12 & -3/4 \end{bmatrix},$$

then find B^4 (where $B^4 = B \cdot B \cdot B \cdot B$).

- A. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ B. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ C. $\begin{bmatrix} -1/6 & -1/2 \\ -1/12 & -3/4 \end{bmatrix}$
 D. $\begin{bmatrix} -9 & 6 \\ 1 & -2 \end{bmatrix}$ E. none of these

16. $\sum_{k=1}^{\infty} 5\left(\frac{e}{\pi}\right)^k = (?)$

- A. $\frac{5\pi}{\pi-e}$ B. $\frac{5e}{\pi-e}$ C. $\frac{5e}{\pi}$ D. 5 E. none of these

17. If $\frac{x^3 + 9x^2 + 11x + 5}{x(x+1)^3} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{(x+1)^3}$, then find $A + B + C + D$.

- A. 0 B. 1 C. -2 D. 2 E. none of these

18. If $n^2 = 10$, then find $\frac{n^{n+4} + n^{n-3}}{n^{n+2} - n^{n-1}}$ in terms of n .

- A. $\frac{2n-7}{3}$ B. 1 C. $\frac{n^2 + n - 12}{n^2 + n - 2}$ D. $\frac{1000n+1}{100n-10}$ E. none of these

19. Solve for x :

$$(2x^2 + 12x + 19)^2 - 4 \cdot (2x^2 + 12x + 19) + 3 = 0$$

- A. 3, 1 B. $-3 \pm \frac{\sqrt{2}}{2}i$ C. -2, -3, -4 D. 1, 2, 3 E. none of these

20. Find the sum in base 10:

$$111_2 + 777_8 + AAA_{12}$$

- A. 1997 B. 2000 C. 2089 D. 2197 E. none of these

21. Find the roots of the equation $x^4 + 4x^3 + 2x^2 + 4x + 1 = 0$, given that $(x+1)^4 = x^4 + 4x^3 + 6x^2 + 4x + 1$.

- A. -1 B. $\pm i, -2 \pm \sqrt{3}$ C. -1, -2, -3, -4
D. $\pm 2, \pm i$ E. none of these

22. Find the area of the quadrilateral with vertices at the foci of

$$16x^2 - 9y^2 - 96x - 36y - 36 = 0$$

and at the endpoints of the major axis of

$$x^2 + 4y^2 - 2x - 16y + 1 = 0.$$

- A. 16 B. 32 C. 18 D. 36 E. none of these

23. Given that:

$$\begin{aligned}x &= 0.\overline{abcdef} & 4x &= 0.\overline{efabcd} \\2x &= 0.\overline{cdefab} & 5x &= 0.\overline{fabcde} \\3x &= 0.\overline{bcdefa} & 6x &= 0.\overline{defabc}\end{aligned}$$

and $a + b + c + d + e + f = 27$, find f .

- A. 7 B. 3 C. 1 D. 5 E. none of these

24. Evaluate:

$$\frac{\prod_{k=2}^{1295} \log_k(k+1)}{\sum_{k=1}^3 \log_2 k}$$

- A. 6 B. $\log(1296)$ C. 4 D. $\log(6)$ E. none of these

25. A fun game to play with Mike D. is to tell him that he has three arms. On any such telling, there are only three possibilities: either he believes you, he does not believe you, or he does not care. Also, on any such telling, there is a $4/5$ probability that he does not believe you and a $1/6$ probability that he does not care. If you tell him that he has three arms seven times, find the probability that he will not believe you 5 times, believe you once, and not care once.

- A. $\frac{256}{140625}$ B. $\frac{4}{5}$ C. $\frac{3584}{46875}$ D. $\frac{2}{25}$ E. none of these

TIEBREAKERS

TB1 Claus, Sevren, Blixa, Cevin, Sascha, and Bill each have \$1 and each is willing to give \$0.40 to one different person. What is the largest amount of money one of them could have?

TB2 Simplify: $\frac{x^3 - x^2 + 2x + 4}{x^3 + 8}$.

TB3 If m and n are the roots of $x^2 + 1997x + 1997$, find $\frac{1}{m} + \frac{1}{n}$.