1997 Hoover High School Math Tournament
Comprehensive Level Ciphering

1.1 Evaluate:

$$\lim_{h \to 0} \frac{f(h+97) - f(97)}{h}$$

If $f(x) = (x+1)^2$.

ANSWER: 196.

1.2 Sets A, B, and C each have 1000 elements. Given that $A \cap B$ has 121 elements and $B \cap C$ has 3 elements, find the maximum number of elements in $(A \cup B) \cap C$.

ANSWER: 882.

1.3 Evaluate the sum:

$$\sum_{n=0}^{\infty} 4\left(\frac{1}{2}\right)^{2n}$$

ANSWER: 16/3.

1.4 Trapezoid ABCD has bases $\overline{AB}$ and $\overline{CD}$ and height h. If $\overline{BC} = h = 4$, $\overline{AB} = 24$, $\overline{CD} = 4\sqrt{37}$, and $\theta = m\angle ADC$, then find $\tan(2\theta)$.

ANSWER: $-\frac{1}{6}$.

1.5 Find the length of the latus rectum of $\frac{x^2}{86} + \frac{(y-13)^2}{\sqrt{43}} = 1$.

ANSWER: $\sqrt{2}$.

2.1 Find the product of all real solutions to:

$$(x^2 + x - 2)^2 - 14(x^2 + x - 2) + 40 = 0.$$

ANSWER: 72.

2.2 Find the value of x that satisfies the system of equations:

$$2x + y + z = 12$$
$$x + 2y + z = 13$$
$$x + y + 2z = 14$$

ANSWER: 9/4.

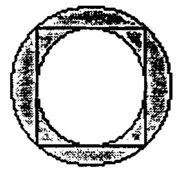
2.3 Charlie, Linus, and Schroeder are racing. The probabilities that Charlie and Linus will win the race are 1/3 and 1/4, respectively. There are no ties. Given that either Charlie or Schroeder will win, find the probability that Schroeder wins.

ANSWER: 5/9.

2.4 Determine the area of the triangle with vertices at (-18, 15); (9, 10); and (4, 1).

ANSWER: 134.

2.5 The picture shows a square of side length l inscribed in a circle and circumscribed about another. Find the area of the shaded annulus.



ANSWER: $\frac{l^2\pi}{4}$.

3.1 <u>The Life and Times of Gore-Vidal Sassoon</u> is a book whose pages are consecutively numbered from 1 to 2000. How many times is the digit 9 printed in numbering the pages?

ANSWER: 600.

3.2 Solve for k:

$$\left(\frac{(k+2)!}{(k+5)!}\right)\cdot\left(\frac{(k+6)!}{(k+1)!}\right)=192$$

ANSWER: 10.

3.3 The third term in the expansion of $(x+3)^{\frac{1}{2}}$ can be expressed as $\mathbf{A}x^{\mathbf{B}}$. Find $\mathbf{A}\cdot\mathbf{B}$.

ANSWER: 27/16.

3.4 If the area and perimeter of an equilateral triangle are numerically equal, find the length of a side.

ANSWER: $4\sqrt{3}$.

3.5 Evaluate:

$$(1+i\sqrt{3})^{10}.$$

ANSWER: $-512\sqrt{3}-512i$.

4.1 $\log\left(\frac{3}{2}\right)+\log\left(\frac{4}{3}\right)+\log\left(\frac{5}{4}\right)+...+\log\left(\frac{2000}{1999}\right)=(?)$.

ANSWER: 3.

4.2 Find the area of the interior of the graph of $x^2-8x+y^2+10y-128=0$.

ANSWER: 169π.

4.3 A regular polygon has 740 diagonals. Find the measure of one of its interior angles.

ANSWER: 171°.

4.4 Where defined,

$$\frac{2\sin\theta\cos^3\theta-2\sin^3\theta\cos\theta}{\sin 4\theta}=(?)$$

ANSWER: ½.

4.5 A Coke™ can is in the shape of a right circular cylinder with a height of 6 inches and a radius of 1 inch. If there are 13.5 calories per cubic inch of Coke™, find the number of calories in half a can of Coke™.

ANSWER: $\frac{81\pi}{2}$.

ALTERNATE 1: Find the sum of all real values of x which satisfy $1997^{x^3+x^2+x+1} = 1$.

ANSWER: -1.

ALTERNATE 2: Find the remainder when 2^{12} is divided by 7.

ANSWER: 1.

ALTERNATE 3: If the sum of the squares of three consecutive positive even integers is 440, find the product of the smallest and the largest.

ANSWER: 140.