1997 Hoover High School
Geometry Written Test

1. The sides of a triangle are x, 2x, and y. In terms of x, between what values must y fall?

a. $2x < y < 3x$ b. $x < y < 2x$ c. $x < y < 3x$ d. $\frac{x}{3} < y < x$ e. none of these

2. A regular hexagon has an area of $48\sqrt{3}$. Find the square of the length of a side of the hexagon.

a. 192 b. $8\sqrt{3}$ c. $4\sqrt{2}$ d. 32 e. none of these

3. A spider is standing at the point (x_0, y_0) in the coordinate plane. She decides to walk west 4 units and then walk north 1 unit. Her new position is (x_1, y_1). Next she walks south 4 units, followed by walking west 1 unit. Her new position is (x_2, y_2). Find the area of the triangle (in square units) with vertices at (x_0, y_0), (x_1, y_1), and (x_2, y_2).

a. 8 b. 8.5 c. 9 d. 9.5 e. none of these

4. If a regular polygon has between 100 and 125 diagonals, then it could have how many sides?

I. 15 III. 17
II. 16 IV. 18

a. II b. III and IV c. II and III d. IV e. none of these

5. Find the area of the region defined by the graph of $x^2 - 8x + y^2 + 6y - 23 \le 0$.

a. 48π b. 23π c. 30π d. 31π e. none of these

6. In triangle ABC, $\overline{AM}$ bisects $\angle CAB$. $\overline{AC} = \frac{50}{3}$, $\overline{MB} = 7$, and $\overline{BA} = 21$. Find the length of $\overline{CM}$.

a. 7 b. $\frac{50}{3}$ c. $\frac{441}{50}$ d. $\frac{50}{9}$ e. none of these

7. A sphere is inscribed in a cube of edge E. Find, in terms of E, the volume of the sphere.

a. $\frac{4E^3\pi}{3}$ b. $\frac{E^3\pi}{3}$ c. $\frac{E^3\pi}{6}$ d. $\frac{E^2\pi}{3}$ e. none of these

8. Find the area of a trapezoid with consecutive side lengths of 50, 37, 50, and 65.

a. 2175 b. 2448 c. 2875 d. 2512 e. none of these

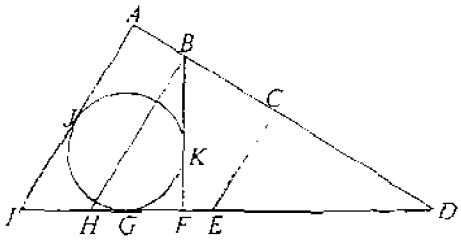
9. Two of the faces of a right rectangular box have area 24, two others have area 15, and the other two have area 40. Find the volume of the box.

a. 90 b. 100 c. 110 d. 120 e. none of these

10. Find the area of the region that is contained in the graph of $|x-4|+|y-4| \le 8$ but is *not* contained in the graph of $(x-4)^2+(y-4)^2 \le 32$.

a. $128-32\pi$ b. $64-16\pi$ c. $32\pi-64$ d. $32\pi-16$ e. none of these

11. In the figure, the circle with center on segment $\overline{BH}$ is tangent to the corresponding lines at K, G, and J. $\angle IAD, \angle HBD, \angle BFD$, and $\angle ECD$ are all right angles. If $\overline{HG}=3$, $\overline{AB}=4$, and $\overline{CD}=9$, find the length of $\overline{EF}$.



a. $\frac{10}{9}$ b. $\frac{41}{36}$ c. $\frac{21}{18}$ d. $\frac{43}{36}$ e. none of these

12. In circle O, $\overline{AB}$ is a diameter. From point P outside the circle, tangent $\overline{PA}$ and secant $\overline{PB}$ are drawn. $\overline{PB}$ intersects the circle at C. If $\overline{PA}=4$ and $\overline{AB}=3$, find the length of $\overline{AC}$.

a. 1.8 b. 2 c. 2.2 d. 2.4 e. none of these

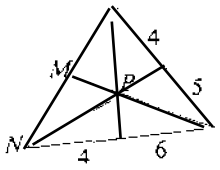
13. Two circles, one of radius 5 and another of radius 8, are in distinct, parallel planes. The perpendicular distance between the two planes is 4 and the distance between the centers of the circles is 4. Every point on the circumference of the smaller circle is connected by a line segment to the closest corresponding point on the circumference of the larger circle. Find the area of the region defined by all such line segments.

a. 65π b. 66π c. 67π d. 68π e. none of these

14. If a polyhedron has 6 faces and 9 edges, then how many vertices does it have?

a. 11 b. 5 c. 9 d. 13 e. none of these

15. The three lines intersect at point P in the interior of the triangle. The distances are as labeled and the length of $\overline{MN}$ is x. If a is the greatest possible value and b is the least possible value such that $a < x < b$, find $a + b$.

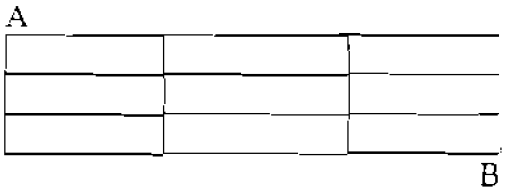


a. 11 b. $\frac{95}{11}$ c. 10 d. $\frac{100}{11}$ e. none of these

16. How many right triangles with integral sides have numerically equal perimeter and area?

a. 0 b. 1 c. 2 d. infinitely many e. none of these

17. The game of Bonkers! is played on a 3 x 3 board as shown. The object of the game is to start at A, go to B, and then return to A. When going from A to B, the player can only move down and to the right. When returning from B to A, the player can only move up and to the left. Following these rules, how many different paths may a player take to win a game of Bonkers!?



a. 81 b. 200 c. 300 d. 400 e. none of these

18. A lattice point is a point in the plane with only integer coordinates (i.e.-(4, 3) is a lattice point, whereas (2.09, -1.4) is not). How many lattice points with non-negative coordinates lie on the graph of $13x + 15y = 273$?

a. 0 b. 1 c. 2 d. 3 e. none of these

19. Find the length of the longest altitude in a triangle with sides of lengths 3, 5, and 6.

a. $\frac{5\sqrt{14}}{3}$ b. $\frac{4\sqrt{14}}{3}$ c. $\frac{4\sqrt{14}}{5}$ d. $\frac{2\sqrt{14}}{3}$ e. none of these

20. Two spheres are inscribed in a cone such that the larger sphere is tangent to the base of the cone and the smaller sphere is tangent to the top of the larger sphere. If the ratio of the radius of the smaller sphere to the radius of the larger sphere is ½, find the ratio of the sum of the volumes of the spheres to the volume of the cone.

a. $\frac{5}{9}$ b. $\frac{9}{16}$ c. $\frac{5}{8}$ d. $\frac{2}{3}$ e. none of these

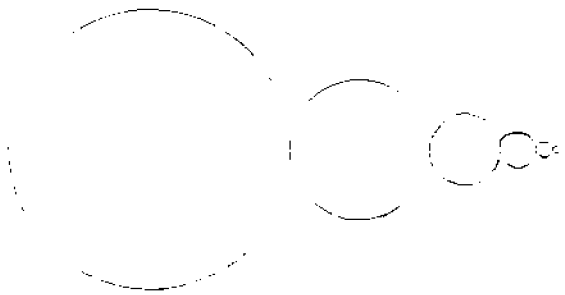
21. Quadrilateral $ABCD$ has an obtuse angle at A. $\overline{DA} = 15$, $\overline{AB} = 4$, $\overline{BC} = 8$, and $\overline{DC} = 15$. If the area of triangle ABD is $6\sqrt{21}$ and the area of the circle circumscribed about triangle BCD is X, then find 4X.

a. 225π b. 256π c. 289π d. 324π e. none of these

22. A triangle has vertices at the origin and the intersection points of the coordinate axes with the line $y = -ax + b$. Find the area of the triangle in terms of a and b.

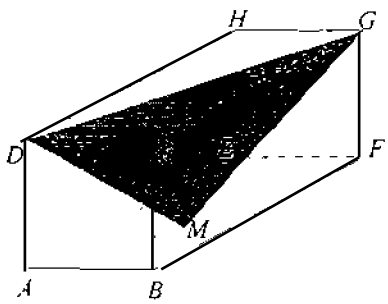
a. $\frac{a^2}{b}$ b. $\frac{a^2}{2b}$ c. $\frac{b^2}{a}$ d. $\frac{b^2}{2a}$ e. none of these

23. Consider the circles below, where the radius of each circle is ½ the length of the radius of the circle directly to its left. If the diameter of the largest circle is 16, then what is the sum of the areas of all six of the circles?



a. $\frac{341\pi}{4}$ b. $\frac{1365\pi}{16}$ c. $\frac{5461\pi}{64}$ d. $\frac{21845\pi}{256}$ e. none of these

24. In the figure, $ABCDEFGH$ is a rectangular box. $\overline{AB} = 10$, $\overline{BC} = 12$, and $\overline{BF} = 20$. Point M is selected on $\overline{EB}$ such that $4\overline{MB} = \overline{EM}$. Find the area of ΔDGM.



a. $60\sqrt{5}$ b. $60\sqrt{6}$ c. $4\sqrt{1158}$ d. $4\sqrt{1159}$ e. none of these

25. A cone is placed on the ground such that its vertex is on the ground and its base is parallel to the ground. The base radius of the cone is 6 and the height of the cone is 18. The cone is filled half-way, by height, with water and then sealed so that no water may leak out. The cone is then turned over and placed on the ground so that its base rests on the ground. Find the new height of the water.

a. 9 b. $9\sqrt[3]{7}$ c. $18 - 9\sqrt[3]{7}$ d. $9\sqrt[3]{7} - 9$ e. none of these

TIEBREAKERS

TB1 How many lines of symmetry does a regular octagon have?

TB2 What is the total surface area of a right circular cone with height 5 and base diameter 24?

TB3 The ratio of an edge of a cube to the radius of a sphere is 1 to 2.5. Find the ratio of their volumes (smallest to largest).