

1998 Hoover High School Math Tournament
Algebra II Ciphering
February 21, 1998

1. Find the product of the two matrices.

$$\begin{bmatrix} 1 & 2 & 3 & \dots & 7 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ \vdots \\ 7 \end{bmatrix}$$

Answer: [140]

2. Given: $2x + 3y = 8$
 $3x - 5y = 6$, find $11x + 7y$.

Answer: 38

3. Find the sum of the solutions to the equation $y - 6 + \sqrt{y} = 0$.

Answer: 4

4. Find the equation of the directrix of $y = 2x^2 - 12x + 29$.

Answer: $y = 8\frac{7}{8}$

5. Find k such that $x + 1$ will be a factor of $x^{79} + kx^2 - 8$.

Answer: 9

6. What is the coefficient of the x^4 term in the expansion of $(x^2 - x^{-\frac{4}{3}})^7$?

Answer: -35

7. Simplify:

$$\sqrt[4]{\sqrt[3]{256}} \cdot \sqrt[5]{\sqrt{32}} \cdot \sqrt[3]{\sqrt[4]{4096}}$$

Answer: 2

8. Given that $f(x) = \frac{ex - \pi}{2x - e}$, find $f^{-1}(2)$.

Answer: $\frac{2e - \pi}{4 - e}$

9. A committee consisting of six girls and four boys is to form a subcommittee made up of four people. What is the probability of selecting a subcommittee consisting of three girls and a boy or one girl and three boys?

Answer: $\frac{52}{105}$

10. Given that $\log 3 = 0.477$, $\log 5 = 0.699$, and $\log 7 = 0.845$, find $\log 42$.
Answer: 1.623

11. Which term of the sequence $-8, -3, 2, 7, \dots$, is 387?

Answer: 80

12. Simplify:

$$\frac{(2 - 3i)(1 + 4i) + (-2 + i)(5 - i)}{(2 + i)(4 - i) + (6i)(i)}$$

Answer: $3 + 2i$

13. How many real zeros does the following function have?

$$f(x) = 8x^7 + 2x^5 + x + 1$$

Answer: 1

14. The square root of A varies inversely with C and directly with B^2 . If $A = 25$ when $B=30$ and $C=10$, find B when $A= 36$ and $C= 3$.

Answer: 18

15. Find the area inside the graph of $9x^2 + 25y^2 = 225$ but outside the graph of $x^2 + y^2 = 9$.

Answer: 6π

16. Express the sum as a reduced improper fraction: $1.\bar{4} + 1.\bar{18} + 0.4\bar{5}$

Answer: $\frac{339}{110}$

17. A certain jacket is discounted 50%, then 40%, and finally 10%. If the jacket was discounted just once, what percent off would be equivalent to the three successive discounts?

Answer: 73%

18. $n!!$ is defined as $n(n-2)(n-4)\cdots(2)$ if n is even and $n(n-2)(n-4)\cdots(1)$ if n is odd. Find $\frac{9!!}{9!}$

Answer: $\frac{1}{384}$

19. Find the determinant of the 4 x 4 matrix:

$$\begin{bmatrix} 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ -1 & 0 & 1 & 2 \end{bmatrix}$$

Answer: 0

20. Find the sum of the x-coordinates when $y=0$ for $y = 2x^3 - 11x^2 + 2x + 15$.

Answer: $\frac{11}{2}$

21. Simplify:

$$\frac{x^{-9} + y^{-9}}{x^{-3} + y^{-3}}$$

Answer: $\frac{x^6 - x^3y^3 + y^6}{x^6y^6}$

22. Given that $g(x) = x^2 - 3$, $h(x) = \sqrt{x}$, $f(x) = x - 20$, and $k(x) = x + 1$, find the value of x such that

$$k(h(f(g(x)))) = x$$

Answer: 12

23. Evaluate: $\sum_{n=1}^5 \left(\frac{2}{3}\right)^{n-1}$

Answer: $\frac{211}{81}$