

1998 Hoover High School Math Tournament
Algebra II Written Test
February 21, 1998

1. Let A = the sum of the roots of $x^3 + 5x^2 - 6x - 16 = 0$, B = the largest root, and C = the product of all three roots. Find $B^A \cdot C$.

- A. 400 B. $\frac{1}{2}$ C. 13 D. $\frac{25}{32}$ E. none of these

2. If $\frac{3x^2 + 2x - 37}{(x-2)(x+5)(x+1)} = \frac{A}{x-2} + \frac{B}{x+5} + \frac{C}{x+1}$, then compute $A + B + C$.

- A. 5 B. 0 C. 3 D. 2 E. none of these

3. The sum of nine consecutive numbers is 918. Find the smallest one of these integers.

- A. 96 B. 98 C. 100 D. 102 E. none of these

4. How many negative roots does the equation

$$8x^8 + 6x^6 + 4x^4 + 2x^2 = 7x^7 + 5x^5 + 3x^3 + x + 1$$

have?

- A. 3 B. 8 C. 1 D. 0 E. none of these

5. $\sum_{n=3}^{29999} \log\left(\frac{n}{n+1}\right) = (?)$

- A. 4 B. 0 C. -2 D. -4 E. none of these

6. Find all real roots of the equation

$$(x^2 + 4x + 5)^4 - 13(x^2 + 4x + 5)^2 + 36 = 0$$

- A. -3, -1 B. $\pm 3, \pm 1$ C. no real roots D. $-3, -1, 2 \pm \sqrt{2}$
E. none of these

7. Solve for x : $\frac{5x}{x-2} = 7 + \frac{10}{x-2}$

- A. -2 B. 2 C. \emptyset D. 2, -2 E. none of these

8. $\frac{25-i}{12-13i} + \frac{10-4i}{7+3i} = (?)$

- A. $2+0i$ B. $\frac{715}{461} + \frac{255}{461}i$ C. $0+2i$ D. $3+2i$
 E. none of these

9. A hexagon has vertices at the points $(-2,5)$, $(1,6)$, $(2,-1)$, $(-3,4)$, $(3,2)$, and $(-2,0)$. Find the area of this hexagon.

- A. $\frac{49}{4}$ B. $\frac{49}{2}$ C. $\frac{57}{2}$ D. 57 E. none of these

10. If $A = \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 3 \\ 3 & -2 \end{bmatrix}$, and $C = A + B$, find C^{-1} .

- A. $\begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix}$ B. $\begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}$ C. $\begin{bmatrix} -\frac{5}{17} & -\frac{3}{17} \\ -\frac{4}{17} & \frac{1}{17} \end{bmatrix}$ D. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
 E. none of these

11. Simplify when $a = 2997$, $b = 4192$, $c = 6$, and $d = 1681$.

$$\frac{a^4 b^3 c^2 d}{b^{\frac{1}{2}} d^2} \cdot \frac{4b^{\frac{3}{2}}}{5cd^2} \cdot \frac{3d^0}{2ab^3 c^2} \cdot \frac{10d^3 (19c)^0}{3a^2 b}$$

- A. 1998 B. 666 C. 382,884,732 D. 20,151,828
 E. none of these

12. Find the values of x that make the expression $\frac{x}{2} - \frac{1}{2x}$ positive.

- A. $(0, \infty)$ B. $(1, \infty)$ C. $(-1, 0) \cup (1, \infty)$ D. $(-\infty, \infty)$
 E. none of these

13. The sum of two numbers is 9 and their product is 9. Find the ratio of the two numbers' harmonic mean to their geometric mean.

- A. 6 B. $\frac{1}{6}$ C. $\frac{3}{2}$ D. $\frac{2}{3}$ E. none of these

14. Let $f(x) = \frac{x}{x-1}$. Find the composition $f^{-1}(f^{-1}(f^{-1}(x)))$.

- A. x B. $\frac{x}{x-1}$ C. $\frac{1}{1-x}$ D. $\frac{1}{x-1}$ E. none of these

15. Bag A contains 3 apples, 2 bananas, and 3 cantaloupes; bag B contains 1 apple, 3 bananas, and 2 cantaloupes; bag C contains 2 apples and 5 bananas. One of the three bags is chosen at random and a piece of food is randomly chosen from it. Find the probability that the food chosen was a banana.

- A. $\frac{41}{84}$ B. $\frac{139}{504}$ C. $\frac{17}{72}$ D. $\frac{1}{2}$ E. none of these

16. x is directly proportional to the sum of the squares of y and z and inversely proportional to y and the square of z . If $x = 8$ when $y = \frac{1}{2}$ and $z = \frac{\sqrt{3}}{2}$, find y when $x = 1$ and $z = 6$.

- A. $2\sqrt{3}$ B. 1 C. 3 D. 6 E. none of these

17. The product of two functions, neither of which is even nor odd, **could be** which of the following?

I. even II. odd III. neither

- A. I B. I and II C. I and III D. I, II, and III E. none of these

18. Find the area of the graph bounded by the x -axis, the y -axis, and the graph of

$$\begin{vmatrix} x & y & 2 \\ 3 & -1 & 0 \\ 2 & 1 & 1 \end{vmatrix} = 0.$$

- A. $\frac{100}{3}$ B. $\frac{50}{3}$ C. $\frac{25}{3}$ D. 15 E. none of these

19. If the vertex of the parabola $2y = 4x^2 + 6x + 3$ is (a, b) , find $a^2 + b^2$.

- A. 3 B. -3 C. $\frac{9}{8}$ D. $-\frac{9}{8}$ E. none of these

20. Find the fifth term in the expansion of $(2x - 1)^9$.

- A. $4032x^5$ B. $-4032x^5$ C. $-2016x^4$ D. $2016x^5$ E. none of these

21. The equation $64x^2 - 36y^2 - 384x - 144y - 432 = 0$ has what shape?

- A. parabola B. hyperbola C. two intersecting lines D. point
E. none of these

22. The ratio of the area of a certain circle to its circumference is 3. If the circle were centered in the coordinate plane at the point $(2, -1)$, then the equation of the circle can be expressed as $Ax^2 + By^2 + Cx + Dy + E = 0$. Find $A + B + C + D + E$.

- A. -31 B. 4 C. -24 D. -36 E. none of these

23. Five numbers (a, b, c, d, e) satisfy the system of equations

$$3a + b + c + d + e = 11$$

$$a + 3b + c + d + e = 13$$

$$a + b + 3c + d + e = 5$$

$$a + b + c + 3d + e = 3$$

$$a + b + c + d + 3e = 17$$

Find $2a - 3b + c - 4d - e$.

- A. -1 B. 5 C. -3 D. -2 E. none of these

24. Two trains are 250 miles apart and a fly is sitting on the nose of one of the trains. The trains start moving toward one another, one at 30 miles an hour and the other at 45 miles an hour, while at the same time the fly starts flying toward the other train at 63 miles an hour. Upon reaching the other train, the fly turns around and flies to the original train, and repeats this until the trains collide. How far had the fly flown between the trains until his untimely demise?

- A. 250 miles B. 200 miles C. 225 miles D. 215 miles E. none of these

25. How many positive integer divisors does the number 5040 have?

- A. 7 B. 60 C. 64 D. 72 E. none of these

TIEBREAKER 1 $\frac{8}{15} + \frac{34}{225} + \frac{152}{3375} + \dots + \frac{3^n + 5^n}{15^n} + \dots = (?)$

TIEBREAKER 2 If $x^{x^{x^{\dots}}} = \sqrt{2}$, then $x = 2^a$. Find a .

TIEBREAKER 3 A dartboard is arranged so that it has three point zones: one is seven points per dart that lands on it, one is five points per dart, and the other is three points per dart. If fifteen darts are thrown at the dartboard, in how many ways can a person score 75 points? (Assume all darts hit the board and no dart can land between point zones.)