1998 Hoover High School Math Tournament
Geometry Examination

1. Find the area of a circle circumscribed about a triangle whose sides have lengths 15, 8, and 17.

a. 50π b. $\frac{225\pi}{4}$ c. 60π d. $\frac{289\pi}{4}$ e. none of these

2. An isosceles trapezoid with base lengths of 16 and 36 is circumscribed about a circle. Find the circumference of the circle.

a. 24π b. 28π c. 32π d. 36π e. none of these

3. Find the area of an equilateral triangle inscribed in the circle defined by the equation

$$(x-1998)^2+(y+1998)^2=25.$$

a. $\frac{75\sqrt{3}}{4}$ b. $\frac{75\sqrt{3}}{2}$ c. $\frac{25\sqrt{3}}{2}$ d. $25\sqrt{3}$ e. none of these

4. In 3-4-5 triangle ABC, calculate the sum of the lengths of the altitudes drawn from vertices A, B, and C.

a. 9 b. 9.4 c. 9.8 d. 10.2 e. none of these

5. A regular hexagon is inscribed in a circle of radius 4. Find the area between the perimeter of the circle and the perimeter of the hexagon.

a. $16\pi-12\sqrt{3}$ b. $16\pi-18\sqrt{3}$ c. $16\pi-24\sqrt{3}$ d. $16\pi-28\sqrt{3}$ e. none of these

6. Find the distance between the points (1998, -2010) and (1993, -1998).

a. 13 b. 15 c. 17 d. 19 e. none of these

7. A regular dodecagon has S sides. Each of its interior angles measures A degrees, and it has D diagonals. Find $3S+2A+D$.

a. 370 b. 380 c. 390 d. 400 e. none of these

8. If (a,b) is a point whose distance from the point $(2,-3)$ is 7, then which of the following is true?

a. $a^2+b^2+6a=4b+36$ b. $a^2+b^2+6b=4a+36$ c. $a^2+b^2=4a+6b+36$
d. $a^2+b^2=4b+6a+36$ e. none of these

9. The circle circumscribed about ΔABC has $\overline{BC}$ as a diameter. If $AB = \frac{1}{13}$ and $AC = \frac{12}{65}$, then what is the circumference of the circle?

a. $\frac{\pi}{5}$ b. $\frac{12\pi}{65}$ c. $\frac{11\pi}{65}$ d. $\frac{2\pi}{13}$ e. none of these

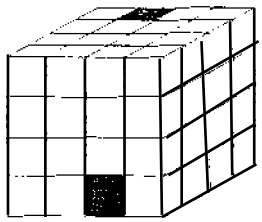
10. Define a translation from point P to point P' such that if $P = (a,b)$, then $P' = (a^2 - 1, b+6)$. If $P = (1,-6)$, then what is the equation of the perpendicular bisector of $\overline{PP'}$?

a. $2x + 12y = 37$ b. $12x + 2y = 37$ c. $12y - 2x = 37$
d. $2x - 12y = 37$ e. none of these

11. Let $f(x) = \frac{2x-3}{3x-2}$. Find the equation of the line through the points $(1, f(1))$ and $(2, f(2))$.

a. $5x - 4y = 9$ b. $5y - 4x = 9$ c. $4y - 5x = 9$ d. $4x - 5y = 9$ e. none of these

12. 64 unit cubes are stacked to form a 4 x 4 x 4 cube. Find the distance between the centers of the two shaded cubes.



a. 4 b. $\frac{\sqrt{102}}{2}$ c. $\sqrt{19}$ d. $4\sqrt{3}$ e. none of these

13. Compute the area of a triangle whose sides have lengths 6, 5, and 9.

a. $10\sqrt{5}$ b. $20\sqrt{5}$ c. $10\sqrt{2}$ d. $20\sqrt{2}$ e. none of these

14. The midpoints of a square are connected consecutively to form a new square. Three of the midpoints of this square are connected consecutively to form a triangle. What fraction of the area of the original square is the area of the triangle?

a. $\frac{1}{16}$ b. $\frac{1}{8}$ c. $\frac{1}{4}$ d. $\frac{\sqrt{2}}{16}$ e. none of these

15. ABCDEF is a regular hexagon with side length 6. Let m be the midpoint of $\overline{EF}$. What is the area of pentagon BCDEm?

a. $28\sqrt{3}$ b. $32\sqrt{3}$ c. $36\sqrt{3}$ d. $40\sqrt{3}$ e. none of these

16. Suppose that the ratio of the base radius to the slant height of a right circular cone is 2:5. If the volume of the cone is V, the **total** surface area is A, and the height is h, then compute the value of $\frac{Ah}{V}$.

a. 10 b. $\frac{21}{2}$ c. 11 d. $\frac{23}{2}$ e. none of these

17. A point is randomly selected inside a square of side 5. Find the probability that it is within 2 units of the center of the square.

a. $\frac{2}{25}$ b. $\frac{2\pi}{25}$ c. $\frac{4}{25}$ d. $\frac{4\pi}{25}$ e. none of these

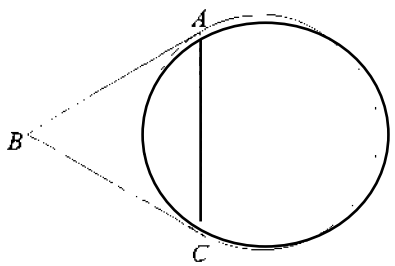
18. All of the faces of a triangular pyramid are isosceles right triangles, except the base, which is an equilateral triangle. The slant height of the pyramid is ℓ. Find the volume of the pyramid.

a. $\frac{\ell^3\sqrt{2}}{3}$ b. $\frac{\ell^2\sqrt{2}}{3}$ c. $\frac{\ell^3\sqrt{3}}{2}$ d. $\frac{\ell^2\sqrt{3}}{2}$ e. none of these

19. In trapezoid $ABCD$, $\overline{AD}\|\overline{BC}$, $3\cdot BC = 2\cdot AD$, F is on $\overline{BA}$, and $\overline{DF}$ bisects $\overline{CA}$. Find the value of $\frac{BF}{BA}$.

a. $\frac{2}{3}$ b. $\frac{1}{2}$ c. $\frac{1}{3}$ d. $\frac{1}{4}$ e. none of these

20. In the picture, the circle is tangent to $\overline{AB}$ and $\overline{BC}$ at points A and C, respectively. If ABC is equilateral and $AB = 6$, then find the area of the circle.

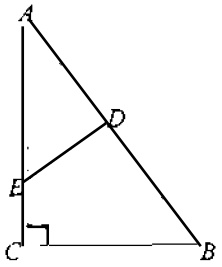


a. 12π b. 16π c. 18π d. 24π e. none of these

21. An ant is standing on the x-axis at the point (4, 0). He has to pick up a crumb at the point (-4, 0), take it to his uncle's wife (Aunt ant) at the point (0, -4), and return home to the point (4, 0). While travelling in the first and third quadrants, the ant cannot enter the interior of the circle $x^2 + y^2 = 16$. Furthermore, the ant is not allowed to walk along the coordinate axes (because the fire ants live there). What is the shortest distance the ant can travel on the entire trip?

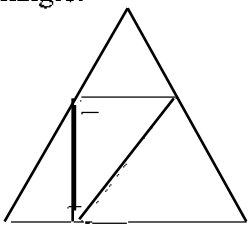
a. 8π b. $8\pi + 4\sqrt{2}$ c. $16\sqrt{2}$ d. $4\pi + 8\sqrt{2}$ e. none of these

22. In the figure, ABC is a right triangle and $\overline{DE} \perp \overline{AB}$. If $AD = 4$, $EC = 2$, and $AB = 12$, then find the perpendicular distance from D to $\overline{BC}$.



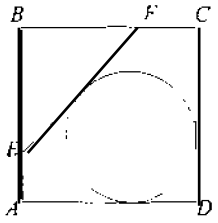
a. 4 b. $\frac{14}{3}$ c. $\frac{16}{3}$ d. 6 e. none of these

23. The picture shows a 9-12-15 right triangle inscribed in an equilateral triangle such that the longer leg is perpendicular to the base of the equilateral triangle. Determine the length of a side of the equilateral triangle.



a. $3\sqrt{3} + 8$ b. $\frac{9\sqrt{3}}{2} + 12$ c. $9 + 8\sqrt{3}$ d. $\frac{9}{2} + 4\sqrt{3}$ e. none of these

24. In the figure, $ABCD$ is a square of side length 8. The circle is tangent to $\overline{CD}$, $\overline{AD}$, and $\overline{EF}$. If $BE = BF = 6$, then find the radius of the circle.



a. $10 - 5\sqrt{2}$ b. $6 - 2\sqrt{2}$ c. $8 - 4\sqrt{2}$ d. $12 - 6\sqrt{2}$ e. none of these

25. A sphere is inscribed in a regular tetrahedron. If an edge of the tetrahedron is 12, then what is the radius of the sphere?

a. $\sqrt{6}$ b. $2\sqrt{6}$ c. $3\sqrt{6}$ d. $4\sqrt{6}$ e. none of these

TIEBREAKERS

TB1 What is the area of a regular octagon with side length $\sqrt{8\sqrt{2}-8}$?

TB2 A solid is formed by rotating a square of side length 6 about one of its diagonals until the vertices of the square return to their initial positions. Find the volume of this solid.

TB3 Find the radius of the circle inscribed in the triangle whose vertices are $(0,0)$, $(5,12)$, and $(5,0)$