Soli Deo Gloria Fall Tournament
Ciphering Round

Written by Billy Dorminy and the Soli Deo Gloria Home Educators
Sponsored by Cyworld Inc. [us.cyworld.com]

September 22, 2007

0.1 Tiebreaker Problem 1

Gracie finds a number so that the sum of its divisors is equal to two times the number. What is the sum of the reciprocals of the divisors of Gracie's number?

0.2 Tiebreaker Problem 2

An angle bisector of a triangle with sides a, b, c, where the bisector goes to side c, has length $\frac{\sqrt{ab((a+b)^2-c^2)}}{a+b}$. Bryan draws a right triangle with integral side lengths. What is the smallest possible length of an angle bisector of his triangle?

0.3 Tiebreaker Problem 3

Joe puts a perfectly spherical Foursquare ball of diameter 5 in a box of dimensions 5 by 5 by 12. What is the diameter of the largest ball that can fit in this box at the same time as Joe's ball?

0.4 Tiebreaker Problem 4

Tyler creates a sequence of positive integers s_i pairwise distinct such that $s_1 = 5$ and for $i > 1$,

$$s_i \mid \sum_{k=0}^{i-1} s_k.$$

Determine the minimum possible value of s_{1000} in Tyler's sequence.

0.5 Tiebreaker Problem 5

Laura drew a triangle with sides of length 2, 3, and 4, such that $AB = 2$, $BC = 3$, and $AC = 4$. She drew a circle S_1 of radius 2 centered at B and another circle S_2 of radius 3 centered at C. Finally, she drew a circle with center D and radius 5 which meets both S_1 and S_2 at right angles.

What is the shortest distance between a point on BC and D?

0.6 Tiebreaker Problem 6

ABC is a triangle of side lengths 12, 13, 15. Given a point P in ABC and points a', b', c' on BC, AC, AB, respectively, such that $\angle Pa'C = \angle Pb'A = \angle Pc'B = 90^\circ$, find the minimum value of $Pa' + Pb' + Pc'$.

0.7 Tiebreaker Problem 7

Consider a set of 6-digit integers S and $b = \frac{3}{2}$. Let $d_0 d_1 \cdots d_5$ be a member of S, where d_i are the digits of the integer, in order, and $d_0 \neq 0$. For every member of S, for any integer $0 \leq i < 6$, the number

$$\sum_{j=0}^{i} d_j \cdot b^{i-j}$$

is integral. Furthermore, for every member of S, for any $i < 5$, the number

$$\sum_{j=0}^{i} d_j \cdot b^{i-j}$$

is divisible by 2.

Find the number of ways to arrange elements of S, not necessarily distinct, in a $6x6$ grid such that every row left-to-right and every column top-to-bottom is a member of S.

0.8 Tiebreaker Problem 8

Find an integer that can be represented as a sum of 2002 positive integers, each with the same sum of their digits, and as a sum of 2003 positive integers, also each with the same sum of their digits.

For instance, 42 can be expressed as the sum of 14 3s or 7 3s and 1 21.