

Soli Deo Gloria Fall Tournament Ciphering Round

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1 Instructions

This round consists of 8 problems, each equally valued, given in pairs. You will have 10 minutes to work on each pair and write your answer on the provided answer sheets. It is recommended that you fill out the name and school fields on your answer sheets before the round begins.

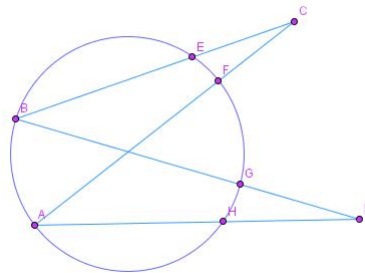
2 Set 1

Problem 1. Consider a variety of paper sizes, A_i for $i \in \mathbb{Z}$ and $i \geq 0$. The size A_0 is defined to be 841 x 1189 mm. For each subsequent size, A_{i+1} is formed by dividing A_i in half with a cut parallel to its shortest side. How large is A_4 paper? Write your answer as [number] x [number]. Answers whose dimensions are within 1 unit of correct will be accepted.

Problem 2. In base 3 there are 3 digits: 0, 1, and 2. Suppose we count in base 3, except we use the digits $\bar{1}$, representing -1, 0, and 1, so that $\bar{1}01$ is equal to $-1 \cdot 3^2 + 0 \cdot 3^1 + 1 \cdot 3^0 = -8$; this method of counting is called *balanced ternary*. Jack takes the positive integers in order from 1, represents them in balanced ternary, and writes the result down. What is the 387th digit he writes down?

Problem 3. If a and b are integers such that $ab = 4014$, what is the probability that $a^{1-b}b^{-1} = 4014^{-1}$?

Problem 4. In the diagram, the measure of $\angle CAD$ is 38 degrees. $4AB = CB$, $4GH = DH$, $AB = 3GH$, and $CA = 3DG$. Find $\angle CBD$.



Problem 5. Find all pairs (m, n) for which $\binom{m}{n} = 2007$.

Problem 6. In triangle ABC , cevians AP , BN , and CM intersect at D . Angles AND and APB have equal measure. Segment AB has length 15, MA has length 6, and AD has length 8. Also, the ratio of CA to BC is 5:6. Find BD .

Problem 7. Randall is running away from a velociraptor. He's running at 15 m/s. The velociraptor is 50 m away from Randall when she spots Randall, and immediately begins chasing him. She accelerates from rest at a constant 10m/s^2 . In how many seconds does the velociraptor catch up to Randall?

Problem 8. In Moczish, a word is made up of 8 binary digits. A word may be transformed by considering any set of digits in a word such that their sum is even and reversing the order of those digits. For instance, in $00\underline{11}0000$, we can apply this transform to the underlined digits and get $0000\underline{11}00$. Words after transformation have the same meaning as before transformation. What is the greatest number of distinct meanings that can be conveyed in Moczish?