Soli Deo Gloria Fall Tournament
Individual Round

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1 About the test:

This is a 15-question, 60-minute exam. Pencils of all colors, pens, protractors, rulers, compasses, scissors, tape, watches, and paper of all sorts are allowed. Genii, calculators, PDAs, calculator watches, rocks, pre-inscribed paper, and anything which is deemed annoying by your proctor are disallowed. If you choose to use scissors or tape, you must do so neatly and throw the results and side effects away before you leave.

2 Scoring:

This round is worth a total of 200 points. The problems are not equally valuable; correct answers are worth from 4 to 25 points. Point values are indicated beside the question number. A wrong or blank answer is worth 0 points.

(4 points) Question 1. Karthik was playing football with his friends. He caught the ball an average of 2.25 times per 15 minutes that he played. If the football game went on for 40 minutes, how many times did he catch the ball?

A) 3 B) 6 C) 9 D) 10 E) 20

(6 points) Question 2. Take three mutually tangent circles of radius 1. A regular hexagon is placed such that the centers of the circles are all vertices of the hexagon. Find the area of the region inside the hexagon but outside the circles.

A) 0 B) $\sqrt{3} - \frac{\pi}{2}$ C) $2\sqrt{3} - \frac{\pi}{3}$ D) $2\sqrt{3} - \pi$ E) $3 - \frac{3\pi}{2}$

(7 points) Question 3. Let s be a sequence of numbers such that $s_0 = s_1 = s_2 = 1$ and, for $n \geq 2$, $s_{n+1} = s_n s_{n-1} - s_{n-2}$. Find

$$\sum_{i=0}^{100} s_i$$

A) 4 B) 8 C) 10 D) 11 E) 75

(10 points) Question 4. There are two distinct right triangles with sides x, $x + 3$, and 20, in some order, for some real numbers x. Find the sum of the lengths of the hypotenuses of these two triangles.

A) $\frac{-3+\sqrt{791}}{2}$ B) $\frac{511}{6}$ C) $\frac{37+\sqrt{791}}{2}$ D) 23 E) $\frac{631}{6}$

(10 points) Question 5. Chuck was working on collecting the complete set of Monty Python episodes when he came across a store sending nothing but Monty Python and Star Wars movies. Unfortunately, this store is magical, much like the Holy Grail, and can only be seen once, and Chuck forgot his credit card, so he can only pay cash. All Monty Python episodes cost the same amount, and all Star Wars movies also cost the same amount. He worked out he could buy 5 Monty Python episodes and 3 Star Wars movies and have 40% of his money left. If he then proceeded to buy 5 more Monty Python episodes he'd have no money left, while if he bought 3 more Star Wars movies, he'd have 5 dollars left. How much money did he start with?

A) 5 B) 6.25 C) 15 D) 25 E) 42

(14 points) Question 6. A pair of integers fulfills all 3 of the following conditions:

1. The sum of the two numbers ends in 3

2. The difference of the two numbers is prime

3. The product of the two numbers is square.

One of the integers in the pair is 9. What is the other?
A) 16 B) 64 C) 4 D) 14 E) 243

(8 points) Question 7. In a math tournament, Miles, Linda, Phillip, Ruby, Jordan, William, Carol, Wesley, Mark, and Brayden were the top 10 students, in some order. Mrs. Poss randomly distributed the 10 trophies to them. The probability that exactly 8 people received the correct trophy may be represented as $\frac{a}{10!}$. Find a.
A) $\binom{10}{2}$ B) $8!\binom{10}{2}$ C) $2 \cdot 2\binom{10}{8}$ D) $2 \cdot 8!\binom{10}{2}$ E) $2\binom{10}{8}$

(12 points) Question 8. The game *H3ar+5* is a 4-person card game. In each round of *H3ar+5* , one player gets at least 13 points, and a total of 26 points are given in each round. For instance, Billy could get 15 points, Danny 6, Tommy 5, and Johnny 0 in one round. The game of *H3ar+5* ends when one player's total score is more than 100 at the end of a round. What is the maximum number of rounds a game of *H3ar+5* could go on?
A) 4 B) 7 C) 15 D) 16 E) 22

(12 points) Question 9. Given that the roots of the equation $x^2 + px + 3 = 0$ are positive and not imaginary, find the maximum value of p.
A) $-2\sqrt{3}$ B) $-\sqrt{3}$ C) $\sqrt{3}$ D) $2\sqrt{3}$ E) $\frac{9}{4}$

(13 points) Question 10. Consider a set of line segments, one each of length 1,2,3,...,9. These may be placed such that the nonagon formed has a circumcircle. How many noncongruent such nonagons are possible?
A) 9! B) 8! C) $\frac{8!}{2}$ D) $\frac{8!}{4}$ E) 7!

(17 points) Question 11. Michael Dirr wants to fill the 'family tree' shown with the integers 1 through 7, inclusive, in such a way that no number has a child greater than it. For instance, if 3 appears in the topmost circle, 4,5,6, and 7 cannot appear as its children. In how many ways can he fill the tree in this way?
A) 45 B) 90 C) 120 D) 720 E) 5040

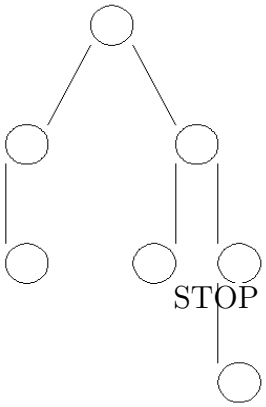

Figure 1: Problem 11: Dr. Dirr's tree.

STOP

(16 points) Question 12. Let $A = (0, 0)$,$B = (0, 6)$, and $C = (0, 8)$. There are two points X where $AX + BX = 10$ and $BX + CX = 10$. What is the y-coordinate of these two points?
A) 2.4 B) 8 C) 6 D) 10 E) 4

(20 points) Question 13. A quadrilateral $ABCD$ has $AB = 12$, $BC = 16$, $CD = 10$, $AD = 10\sqrt{3}$, and can be inscribed in a circle with center O. Find the area of $\triangle ABO + \triangle CDO$.
A) 44 B) $24 + \frac{25}{2}\sqrt{3}$
C) 42 D) $44 + \frac{55}{2}\sqrt{3}$ E) $48 + 25\sqrt{3}$

(23 points) Question 14. An equilateral triangle ABC is inscribed in a circle of radius $\frac{10}{\sqrt{3}}$. Points D and E are on sides AB and AC such that AD/AB=1/2 and AE/AC=4/5. BE and CD intersect at point P. AP is extended to meet BC at F and the circle at G. The length of AG may be represented as $\frac{a}{\sqrt{b}}$, where $\gcd(a, b) = 1$. Find $a + b$.
A) 29 B) 31 C) 43 D) 71 E) 111

(25 points) Question 15. A possibly-degenerate triangle has sides of length a, b, c and total side length of 1. Find the greatest k such that

$$(a - \frac{b}{2})(a - \frac{c}{2}) + (b - \frac{a}{2})(b - \frac{c}{2}) + (c - \frac{a}{2})(c - \frac{b}{2}) \geq k$$

for all a, b, c.
A) $\frac{1}{12}$ B) 1 C) $\frac{11}{9}$ D) $\frac{3}{2}$ E) 2