1. Compute: $\sqrt{2000 \cdot 2007 \cdot 2008 \cdot 2015 + 784}$.

2. The points A, B, C, and D lie in that order on a line. Point E lies in a plane with A, B, C, and D such that $\angle BEC = 78^\circ$. Given that $\angle EBC > \angle ECB$, $\angle ABE = 4x + y$, and $\angle ECB = x + y$, compute the number of positive integer values that y can take on.

3. Back in 1802, the town idiot of Boratsville, Thungar the Stenchified, bought some apples from a vendor at the market. He paid 12 cents for the apples, but when he saw how small the apples were, he demanded two extra apples for free, and the vendor agreed. Ultimately, the price Thungar paid was one penny less per dozen apples than he would have paid had he not received the extra apples. How many apples did Thungar purchase (including the two extra)?

4. Find the smallest positive integer N such that the product $19999N$ ends in the four digits 2007.

5. How many positive 5-digit odd integers are palindromes?

6. If r is a root of $x^2 + x + 6$, then compute the value of

$$r^3 + 2r^2 + 7r + 17.$$

7. Find the number of <u>distinct</u> integers in the list

$$\left\lfloor \frac{1^2}{2007} \right\rfloor, \left\lfloor \frac{2^2}{2007} \right\rfloor, \left\lfloor \frac{3^2}{2007} \right\rfloor, \left\lfloor \frac{4^2}{2007} \right\rfloor, \cdots, \left\lfloor \frac{2007^2}{2007} \right\rfloor,$$

   where $\lfloor x \rfloor$ represents the greatest integer less than or equal to x.

8. Krugman is a little odd. Really odd actually. He has a collection of Massachusetts state quarters and New Hampshire state quarters. All 29 of his Massachusetts quarters are kept face up, while all 11 of his New Hampshire quarters are kept face down on display in his office. Eric, being the prankster that he is, sneaks into Krugman's office and randomly turns over 20 of Krugman's quarters. Find the expected number of Krugman's quarters that are heads up when Eric is finished.

9. Let $F_1 = F_2 = 1$, and $F_{n+2} = F_{n+1} + F_n$ for $n \geq 1$. Find the value of k such that $x = F_k$ is the x-coordinate of the intersection between the linear equations

$$F_{2007}x + F_{2008}y = F_4,$$
$$F_{2008}x + F_{2009}y = -F_3.$$

10. Andrew and Patri each have a deck of cards. Andrew draws two cards from his deck, both of which turn out to be queens. Patri draws two cards from his deck, one of which is an ace, and the other of which is a king. Andrew and Patri then put the remainder of their decks (50 cards left in each deck) down on a table and race to the nearest river. Meanwhile, Chelsea comes along and picks up one of the decks at random. She randomly pulls two more cards out of that deck. It turns out Chelsea's cards make a pair (both of the same rank, i.e., both are 2's, both are 3's, etc.). Find the probability that she drew those two cards from Andrew's deck.

11. In how many distinct ways can a rectangular 3×17 grid be tiled with 17 non-overlapping 1×3 rectangular tiles?

12. If $\omega^{2007} = 1$ and $\omega \neq 1$, then evaluate

$$\frac{1}{1+\omega} + \frac{1}{1+\omega^2} + \frac{1}{1+\omega^3} + \cdots + \frac{1}{1+\omega^{2007}}.$$

Express your answer as a fraction in lowest terms.

13. Before he gets out of bed every morning, Calvin the Compulsive plays a game with a fair coin. He flips it until *either* he flips four consecutive heads *or* he flips six consecutive tails, then he immediately gets out of bed and brushes his teeth. If his last flip is a head, he eats two melons for breakfast. Otherwise, he eats just one. Find the probability that Calvin ate two melons for breakfast this morning.

14. Find the sum of the *real* roots of

$$x^6 + 145x^4 - 2007x^3 + 145x^2 + 1 = 0.$$

15. Let P be a point inside isosceles right triangle ABC such that $\angle C = 90^\circ$, $AP = 5$, $BP = 13$, and $CP = 6\sqrt{2}$. Find the area of ABC.

Note: A complete solutions guide with full explanations and answers to these problems can be found at `www.mistacademy.com/MathDocs/AlabamaARMLtst2007Sol.pdf`**.**