This document is a sample of the MIST Academy *Math Explorers* curriculum for elementary school students. This curriculum is developed to guide students through explorations of fundamental principles of mathematics and problem solving. Whenever possible, games and puzzles are used to entice exploration of new ideas. Many of the lessons are similar to the curriculum promoted by the *Math Olympiads for Elementary and Middle Schools* (MOEMS) program.

# Arithmetic

The MIST Academy curriculum includes not only the regular processes of arithmetic, but also an ability to interpret arithmetic in ordinary language, and an understanding of patterns that illuminate the art of arithmetic.

1. What is the value of $5(3 \cdot 2) - 3(2 \cdot 5)$? *(MATHCOUNTS)*

2. Suppose that you buy a rare stamp for $15, sell it for $20, buy it back for $25, and finally sell it for $30. How much money do you make or lose in buying and selling this stamp?

3. Janet, Joe, and Johnny each collected money for charity. If Janet collected fifty dollars, Joe collected seventy-two dollars, and Johnny collected forty-six dollars, how much money did they collect altogether?

4. James had thirty dollars. Then he spent twelve dollars on groceries, and another six dollars on a book. How many dollars did James have left?

5. In a jar of 960 jelly beans, $\frac{1}{6}$ of the jelly beans are red, and $\frac{1}{8}$ of the jelly beans are green.
    - How many of the jelly beans are red?
    - How many of the jelly beans are green?
    - How many of the jelly beans are either red or green?

6. An Olympiad team is made up of students from the 4th, 5th, and 6th grades only. Seven students are 5th graders, eleven students are 6th graders, and one-third of the entire team are 4th graders. How many students are on the team? *(MOEMS)*

7. Find two prime numbers that have a sum of 28.

8. What simple fraction is equal to the complex fraction? *(MOEMS)*

$$\frac{1}{4 + \frac{1}{2 + \frac{1}{3}}}$$

9. Two dogs run around a circular track 300 feet long. One dog runs at a steady rate of 15 feet per second, the other at a steady rate of 12 feet per second. Suppose they start at the same point and time. What is the least number of seconds that will elapse before they are again together at the starting point? *(MOEMS)*

# Algebra Basics

The Math Explorers curriculum will walk elementary school students gradually through an understanding of the concept of a variable, and how to use variables in basic ways.

1. If $x = 3$, what is the value of $2x + 3$? *(MATHCOUNTS)*

2. Jon's age is five years more than three times Julie's age. If Julie is nine years old, how old is Jon?

3. If there are twice as many gorillas as elephants at the zoo, and those gorillas and elephants have a total of 48 legs, how many elephants are there at the zoo?

4. If $a = 3$ and $b = 2$, find the value of
$$\frac{ab + 2a + 3b}{a - b}.$$

5. Two apples and three oranges cost \$0.71. Three apples and two organges cost \$0.69. How much does just one apples and one orange cost?

6. If 6 is placed at the right end of a two-digit number AB, the value of the three-digit number thus formed is 294 more than AB. What is the original two-digit number AB? *(MOEMS)*

7. The sum of the ages of Al and Bill is 25; the sum of the ages of Al and Carl is 20; the sum of the ages of Bill and Carl is 31. Who is the oldest of the three and how old is he? *(MOEMS)*

8. If a basketball weighs $10\frac{1}{2}$ ounces plus half its own weight, how much does it weigh? *(Martin Gardiner)*

# Counting

Counting techniques start simple, but soon, at the middle school level, their complexity becomes suddenly more intense. The Math Explorers curriculum includes games, problems, and projects that help each student grasp the most important principles.

1. There are ten students in Mrs. Wright's first period class. One day, as an assignment, each student writes a different math problem to challenge every other student in class. For instance, Sarah writes a different problem to challenge Eddie from the one she writes to challenge Marie. How many problems do the students write in total?

2. Many whole numbers between 10 and 1,000 have 2 or 7 as the units digit. How many such numbers are there between 10 and 1,000? *(MOEMS)*

3. Of the thirty aliens living on planet Turing, 20 wear glasses, 16 wear hats, and 8 wear both glasses and hats. How many of the aliens on planet Turing wear neither glasses nor hats?

4. 3, 6, 9, 12, ... are some multiples of 3. How many multiples of 3 are there between 10 and 226? *(MOEMS)*

5. There are 203 students in 10 classrooms at Hogwarts Middle School. Then the largest class must contain at least what number of students?

6. In how many ways can 17 cents in change be made using pennies, nickels, and dimes?

7. How many digits does it take to write the first 200 positive integers?

8. 64 teams play in a single-elimination basketball tournament until only one team remains (and is crowned the champions). How many games are played during the tournament?

# Games and Puzzles

Games and puzzles excite both curious and competitive minds and often lead talented students to a deeper understanding of mathematics. The following are just a few of the games and puzzles that will be explored in the MIST Academy classroom.

1. Shauna did a number trick with Zach. She told him to pick an even number, double it, add 48, divide by 4, subtract 7, multiply by 2, and subtract his original number. She then told him the result he shOuld have attained. What was it? *(MATHCOUNTS)*

2. Begin with a pile of 26 sticks. Two students take turns removing either 1 or 2 sticks from the pile. The game lasts until there are no sticks left, and the student who did not take the last stick wins.

3. In how many ways can 16 rocks be divided into 5 piles such that the total number of rocks in each pile is odd?

4. A farmer is to ferry across a river a goat, a cabbage, and a wolf. Besides the farmer himself, the boat allows him to carry only one of them at a time. Without supervision, the goat will gobble the cabbage whereas the wolf will not hesitate to feast on the goat.

5. Two friends who have an eight-quart jug of water wish to share it evenly. They also have two empty jars, one that holds five quarts, and another that holds three quarts. How can they each measure exactly 4 quarts of water?

6. The king intends to build six fortresses in his realm and to connect each pair by a road. Draw a diagram of the fortress and roads to that there would be exactly three intersections and exactly two roads would cross at each intersection. *(Leningrad Mathematical Olympiad)*

7. Suppose you write out all the integers from 2 to 120. Then you cross out all the multiples of 2, all the multiples of 3, all the multiples of 5, and then all the multiples of 7. What is true about *all* of the remaining numbers in the list?

8. Two players play the following game on a $9 \times 9$ chessboard. They write in succession one of two signs in any free square of the board; the player making the first move writes a plus sign and the other player writes a minus sign. What all the squares of the board are filled, the scores of the players are evaluated. The number of rows and columns containing more plus signs than minus signs is the score of the first player, and the number of all other rows and columns is the score of the second player. What si the highest number of points the first player can gain in an errorless game? *(Leningrad Mathematical Olympiad)*

# Geometry

A solid foundation in geometry begins with the exploration of shapes and sizes, and moves into exploration of the properties of one-, two-, and three-dimensional objects.

1. Farmer Tim keeps his cows in a rectangular pen, one side of which is a wall of his barn. The other three sides have lengths 80 feet, 100 feet, and 80 feet. What is the area of land inside the pen?

2. How many $1 \times 1 \times 2$ blocks fit snuggly inside a $4 \times 8 \times 8$ box?

3. A triangle has a height of 7 cm and a base of 10 cm. What is the area of the triangle in square centimeters?

4. If you draw a circle, a triangle, and a square on a piece of paper, what is the greatest number of points of intersection that could result?

5. A square and a triangle have equal perimeters. The lengths of the three sides of the triangle are 6.1 cm, 8.2 cm and 9.7 cm. What is the area of the square in square centimeters?

   (A) 24 (B) 25 (C) 36 (D) 48 (E) 64

   *(AMC 8)*

6. The length of a rectangle is decreased by three feet, and the width is increased by one foot, forming a square region having an area of 25 square feet. What is the area of the original rectangular region? *(MATHCOUNTS)*

7. On most days, I tied my dog Rover's leash to a post in my yard. His leash is long enough that he can roam around within the limits of a circle with area 300 square feet. One day, I walk Rover to my barn where I tie his leash to the outside corner of the barn. The walls of the barn form a square, and each side of the barn is much longer than Rover's leash. Assuming there are no structures to get in his way, what is the area (in square feet) of the area in which Rover can roam outside the barn?

# Number Sense

Many teachers erroneously dismiss number sense as a collection of worthless tricks. Poorly understood and taught, this might be the case. At MIST Academy we help students see the patterns, arithmetic, algebra, and number theory behind methods that make nearly all of computation, math, and problem solving easier.

1. Find the value of $2(98) + 3(99) + 4(100)$.

2. How many quarters make \$3.25?

3. Find the product of 17 and 25.

4. Compute $64 \cdot 5 - 24 \cdot 5$. (Hint: Think about each product as the value of a pile of nickels.)

5. How many digits are in the product of 27 and $10^{23}$? *(MATHCOUNTS)*

6. July 17, 1977 was a Sunday. What day of the week was July 17, 2007?

7. A box contains over 100 marbles. The marbles can be divided into equal shares among 6, 7, or 8 children with 1 marble left over each time. What is the least number of marbles that the box can contain? *(MOEMS)*

8. If a natural number is multiplied by itself, the result is called a perfect square. Thus, 1, 4, 9, 16, 25, 36, 49, ... are perfect squares and also consecutive because they follow in order. The number 1000 is between two consecutive perfect squares. Which one of these two squares is closer to 1000? *(MOEMS)*

9. The addition below is incorrect. What is the largest digit that can be changed to make the addition correct?

$$\begin{array}{cccc} & 6 & 4 & 1 \\ & 8 & 5 & 2 \\ + & 9 & 7 & 3 \\ \hline 2 & 4 & 5 & 6 \end{array}$$

*(AHSME)*

# Probability

For even many middle school, high school, and even college students, probability is one of the toughest areas of mathematics to understand and master. The Math Explorers curriculum includes many games, puzzles, and problems that gently introduce elementary school students to the concepts of chance, frequency, and ratio as they are used in probability.

1. In a standard deck of 52 playing cards, each card has two properties: rank and suit. There is one card of each rank in each suit, and the suits are clubs ($\clubsuit$), diamonds ($\diamondsuit$), hearts ($\heartsuit$), and spades ($\spadesuit$). The ranks are Ace (**A**), the integers from 2 through 10 inclusive (**2, 3, . . . , 10**), Jack (**J**), Queen (**Q**), and King (**K**):

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A$\clubsuit$ | 2$\clubsuit$ | 3$\clubsuit$ | 4$\clubsuit$ | 5$\clubsuit$ | 6$\clubsuit$ | 7$\clubsuit$ | 8$\clubsuit$ | 9$\clubsuit$ | 10$\clubsuit$ | J$\clubsuit$ | Q$\clubsuit$ | K$\clubsuit$ |
| A$\diamondsuit$ | 2$\diamondsuit$ | 3$\diamondsuit$ | 4$\diamondsuit$ | 5$\diamondsuit$ | 6$\diamondsuit$ | 7$\diamondsuit$ | 8$\diamondsuit$ | 9$\diamondsuit$ | 10$\diamondsuit$ | J$\diamondsuit$ | Q$\diamondsuit$ | K$\diamondsuit$ |
| A$\heartsuit$ | 2$\heartsuit$ | 3$\heartsuit$ | 4$\heartsuit$ | 5$\heartsuit$ | 6$\heartsuit$ | 7$\heartsuit$ | 8$\heartsuit$ | 9$\heartsuit$ | 10$\heartsuit$ | J$\heartsuit$ | Q$\heartsuit$ | K$\heartsuit$ |
| A$\spadesuit$ | 2$\spadesuit$ | 3$\spadesuit$ | 4$\spadesuit$ | 5$\spadesuit$ | 6$\spadesuit$ | 7$\spadesuit$ | 8$\spadesuit$ | 9$\spadesuit$ | 10$\spadesuit$ | J$\spadesuit$ | Q$\spadesuit$ | K$\spadesuit$ |

Note that there are 13 cards in each suit representing the 13 ranks. There are 4 cards of each rank, one of each of the 4 suits. Suppose a standard deck of 52 playing cards is shuffled many times so that the cards are sorted in a stack randomly.

(a) One card is drawn. Find the probability that it is a heart ($\heartsuit$).
(b) One card is drawn. Find the probability that it is a spade ($\spadesuit$).
(c) One card is drawn. Find the probability that it is either a club ($\clubsuit$) or a ($\spadesuit$).
(d) One card is drawn. Find the probability that it is a 4.
(e) One card is drawn. Find the probability that it is a 3 or a heart or both.
(f) Two different cards are drawn. Find the probability that they share the same suit.
(g) Two different cards are drawn. Find the probability that they are both hearts.

2. Jason rolls a pair of regular 6-sided dice.

(a) Find the probability that the sum of the dots on the top of the two dice is 10.
(b) Find the probability that the sum of the dots on the top of the two dice is *at least* 10.

3. Three numbers are randomly selected, with replacement, from the set of integers 1 through 100 inclusive. What is the probability that the product of the three numbers selected will be even? *(MATHCOUNTS)*

# Sequences & Series

Problems involving sequences and series can challenge a student to apply master of arithmetic, pattern hunting, algebra, number theory, and sometimes other areas of mathematics. These problems often frustrate even talented solvers, but can be the most enjoyable at the same time.

1. Find the next three numbers in each of the following sequences:
   (a) 3, 6, 9, 12, 15, 18, __, __, __, ...
   (b) 4, 7, 10, 13, 16, 19, __, __, __, ...
   (c) 4, 8, 12, 16, 20, 24, __, __, __, ...
   (d) 3, 6, 12, 24, 48, 96, __, __, __, ...
   (e) 3, 7, 15, 31, 63, 127, __, __, __, ...

2. Consecutive odd numbers are odd numbers that differ by 2 and follow in order such as 1, 3, 5, 7, 9, or 17, 19, 21. Find the first of seven consecutive odd numbers if the average of the seven numbers is 41. *(MOEMS)*

3. When the 171st even natural number is subtracted from the 219th odd natural number, the result is $z$. Find $z$. *(MATHCOUNTS)*

4. What is the sum of the digits of the first 100 positive even numbers? *(MOEMS)*

5. (Martin Gardiner) Al wanted his father to give him an allowance of $1.00 a week, but his father refused to go higher than 50 cents. After they had argued about it for a while, Al (who was pretty smart in arithmetic) said:

   "Tell you what, Dad. Suppose we do it this way. Today is the first of April. You give me a penny today. Tomorrow give me two pennies. The day after tomorrow, give me four pennies. Each day, give me twice as many pennies as you did the day before."

   "For how long?" asked Dad, looking wary.

   "Just for the month of April," said Al. "Then I won't ask you for any more money for the rest of my life."

   "Okay," Dad said quickly. "It's a deal!"

   Which of the following figures do you think comes the closest to the amount of money that Dad will have to pay Al during the month of April?

   $1
   $10
   $100
   $1,000
   $10,000
   $100,000
   $1,000,000
   $10,000,000

6. Take a look at the counting grid below:

| | | | | | |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 |
| 7 | 8 | 9 | 10 | 11 | 12 |
| 13 | 14 | 15 | 16 | 17 | 18 |
| 19 | 20 | 21 | 22 | 23 | 24 |
| 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 |
| 37 | 38 | 39 | 40 | 41 | 42 |
| 43 | 44 | 45 | 46 | 47 | 48 |
| 49 | 50 | 51 | 52 | 53 | 54 |
| 55 | 56 | 57 | 58 | 59 | 60 |

(a) Start counting up from 1. Circle every third integer. What pattern do you see?

(b) Start counting up from 1. Circle every fourth integer. What pattern do you see?

(c) Start counting up from 1. Circle every fifth integer. What pattern do you see?

(d) What other patterns do you see?

7. See what kinds of patterns you can find below:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| row 1: | 1 | 2 | 4 | 7 | 11 | 16 | 22 | 29 | 37 | 46 |
| row 2: | 3 | 5 | 8 | 12 | 17 | 23 | 30 | 38 | 47 | |
| row 3: | 6 | 9 | 13 | 18 | 24 | 31 | 39 | 48 | | |
| row 4: | 10 | 14 | 19 | 25 | 32 | 40 | 49 | | | |
| row 5: | 15 | 20 | 26 | 33 | 41 | 50 | | | | |
| row 6: | 21 | 27 | 34 | 42 | 51 | | | | | |
| row 7: | 28 | 35 | 43 | 52 | | | | | | |
| row 8: | 36 | 44 | 53 | | | | | | | |
| row 9: | 45 | 54 | | | | | | | | |
| row 10: | 55 | | | | | | | | | |

8. Is it possible to write the natural numbers $1, 2, \ldots, 100$ in a row so that the difference between any two adjacent numbers is no less than 50? *(Leningrad Mathematical Olympiad)*