This document is a sample of the MIST Academy curriculum for middle school students. Your child/student may find some of these problems easy and others very difficult. Students are working with the right curriculum when they get some problems and struggle with others.

Arithmetic

The MIST Academy curriculum includes not only the regular processes of arithmetic, but also an ability to interpret arithmetic in ordinary language, and an understanding of patterns that illuminate the art of arithmetic.

1. What is the value of $5(3 \cdot 2) - 3(2 \cdot 5)$? *(MATHCOUNTS)*

2. Janet, Joe, and Johnny each collected money for charity. If Janet collected fifty dollars, Joe collected seventy-two dollars, and Johnny collected forty-six dollars, how much money did they collect altogether?

3. James had thirty dollars. Then he spent twelve dollars on groceries, and another six dollars on a book. How many dollars did James have left?

4. In a jar of 960 jelly beans, $\frac{1}{6}$ of the jelly beans are red, and $\frac{1}{8}$ of the jelly beans are green.
   - How many of the jelly beans are red?
   - How many of the jelly beans are green?
   - How many of the jelly beans are either red or green?

5. How many digits are in the product of 27 and 10^{23}? *(MATHCOUNTS)*

6. If M is 30% of Q, Q is 20% of P, and N is 50% of P, then $M/N =$

   $$\text{(A) } \frac{3}{250} \qquad \text{(A) } \frac{3}{25} \qquad \text{(A) } 1 \qquad \text{(A) } \frac{6}{5} \qquad \text{(A) } \frac{4}{3}$$

   (AHSME)

7. Fill in the blanks below using the digits 6, 7, 8, and 9 each exactly once in order to create the greatest possible product:

   $$\_\,\_ \times \_\,\_$$

8. Find the smallest possible value of $a + b + c$ where a, b and c are different positive integers that satisfy the following equation:

   $$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{7}{10}$$

   (MATHCOUNTS)

9. Take a look at the counting grid below:

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60

(a) What pattern is made by the multiples of 3?

(b) What pattern is made by the multiples of 4?

(c) What pattern is made by the multiples of 5?

(d) What other patterns do you see?

10. See what kinds of patterns you can find below:

row 1:	1	2	4	7	11	16	22	29	37	46
row 2:	3	5	8	12	17	23	30	38	47	
row 3:	6	9	13	18	24	31	39	48		
row 4:	10	14	19	25	32	40	49			
row 5:	15	20	26	33	41	50				
row 6:	21	27	34	42	51					
row 7:	28	35	43	52						
row 8:	36	44	53							
row 9:	45	54								
row 10:	55									

Algebra

MIST Academy curriculum covers all the topics in a standard pre-algebra and algebra I course. It also includes a host of problem solving techniques and a particular focus on the art of translating words into math. Students who learn algebra as fluently as a language are at a great advantage when it comes to applying their knowledge.

1. If $x = 3$, what is the value of $2x + 3$? *(MATHCOUNTS)*

2. If there are twice as many gorillas as elephants at the zoo, and those gorillas and elephants have a total of 48 legs, how many elephants are there at the zoo?

3. The sum of the digits of a positive two-digit integer is 45 less than the integer. What is the tens digit of the integer? *(MATHCOUNTS)*

4. Find the value of $a + b$ given that (a, b) is a solution to the system

$$\begin{aligned} 3a + 7b &= 1977, \\ 5a + b &= 2007. \end{aligned}$$

   (iTest)

5. If a basketball weighs $10\frac{1}{2}$ ounces plus half its own weight, how much does it weigh? *(Martin Gardiner)*

6. Alice is twice as old as Judy, who is younger than Alex by 3 years. If the sum of their ages is 27, how old is Alice?

7. The largest of three numbers is 10 more than the smallest. The middle number is the average of the other two. If the sum of the three numbers is 42, what are the three numbers?

8. The sum of two numbers is 10, and their product is 21. Find the sum of their squares.

9. If each boy purchases a patty and each girl purchases a bun, they would spend a total of one cent more than if each boy purchased a bun and each girl purchased a patty. We know that the number of boys is greater than the number of girls. What is the difference between the number of boys and the number of girls? *(Leningrad Mathematical Olympiad)*

Counting

Counting techniques includes casework, combinations, complementary counting, the "greedy algorithm", permutations, the principle of inclusion-exclusion, and the use of Venn diagrams.

1. How many three-digit numbers are divisible by 13?

   (A) 7 (B) 67 (C) 69 (D) 76 (E) 77

   (AMC 8)

2. Of the thirty aliens living on planet Turing, 20 wear glasses, 16 wear hats, and 8 wear both glasses and hats. How many of the aliens on planet Turing wear neither glasses nor hats?

3. Of the pet owners who bought pet food at Bob's Pet Shop on Saturday, each of the following is true:

   - 10 own at least one cat, at least one dog, and at least one turtle.
   - 15 own at least one cat and at least one dog.
   - 17 own at least one cat and at least one turtle.
   - 13 own at least one dog and at least one turtle.
   - 28 own at least one cat.
   - 21 own at least one dog.
   - 24 own at least one turtle.
   - 2 own no cat, no dog, and no turtle.

   How many pet owners bought pet food at Bob's Pet Shop on Saturday?

4. Using quarters, dimes, nickels, and pennies, find the smallest number of coins that could equal each of the following amounts:

   (a) \$.37 (c) \$1.32
   (b) \$.73 (d) \$3.41

5. How many digits does it take to write the first 2007 positive integers?

6. In how many ways can 47 cents in change be made using quarters, dimes, nickels, and pennies?

7. A large cube is painted green and then chopped up into 64 smaller congruent cubes. How many of the smaller cubes have at least one face painted green? *(Vestavia ARML TST)*

8. Every student in a 2^{nd} grade class sends a valentine to each of the other students in the class, for a total of 306 valentines. How many students are in the class? *(MATHCOUNTS)*

9. Alice, Beth, and Carlie have ten identical red balloons. In how many ways can the girls split up the balloons such that each of them gets at least one red balloon?

Games and Puzzles

While many educators are skilled at the use of games and puzzles in the classroom, games and puzzles may be the single most under-appreciated area of pre-college mathematics. Games and puzzles excite both curious and competitive minds and often lead talented students to a deeper understanding of mathematics. The following are just a few of the games and puzzles that will be explored in the MIST Academy classroom.

1. Shauna did a number trick with Zach. She told him to pick an even number, double it, add 48, divide by 4, subtract 7, multiply by 2, and subtract his original number. She then told him the result he shOuld have attained. What was it? *(MATHCOUNTS)*

2. Begin with a pile of 26 sticks. Two students take turns removing either 1 or 2 sticks from the pile. The game lasts until there are no sticks left, and the student who did not take the last stick wins.

3. A farmer is to ferry across a river a goat, a cabbage, and a wolf. Besides the farmer himself, the boat allows him to carry only one of them at a time. Without supervision, the goat will gobble the cabbage whereas the wolf will not hesitate to feast on the goat.

4. Aliquot part is another name for a proper divisor, i.e. any divisor of a given number other than the number itself. A prime number has only one aliquot part - the number 1. 1, 2, 3, 4, 6 are all aliquot parts of 12. The number 1 does not have aliquot parts. In the Aliquot game, players take turns subtracting an aliquot part of the number left by their opponent. The winner is the last player able to perform such a subtraction. The loser is the player left with a number that has no aliquot parts - 1. Thus the objective of the game is to leave your opponent without a move.

5. Two friends who have an eight-quart jug of water wish to share it evenly. They also have two empty jars, one that holds five quarts, and another that holds three quarts. How can they each measure exactly 4 quarts of water?

6. The king intends to build six fortresses in his realm and to connect each pair by a road. Draw a diagram of the fortress and roads to that there would be exactly three intersections and exactly two roads would cross at each intersection. *(Leningrad Mathematical Olympiad)*

7. Two players play the following game on a 9×9 chessboard. They write in succession one of two signs in any free square of the board; the player making the first move writes a plus sign and the other player writes a minus sign. What all the squares of the board are filled, the scores of the players are evaluated. The number of rows and columns containing more plus signs than minus signs is the score of the first player, and the number of all other rows and columns is the score of the second player. What si the highest number of points the first player can gain in an errorless game? *(Leningrad Mathematical Olympiad)*

Geometry

1. A square and a triangle have equal perimeters. The lengths of the three sides of the triangle are 6.1 cm, 8.2 cm and 9.7 cm. What is the area of the square in square centimeters?

   (A) 24 (B) 25 (C) 36 (D) 48 (E) 64

   (AMC 8)

2. The length of a rectangle is decreased by three feet, and the width is increased by one foot, forming a square region having an area of 25 square feet. What is the area of the original rectangular region? *(MATHCOUNTS)*

3. Find the length of the median to the hypotenuse of a right triangle whose legs have lengths 10 and 24.

4. The vertices of triangle ABC are $A(1, -2)$, $B(4, 0)$ and $C(2, 2)$. What is the area of triangle ABC? *(MATHCOUNTS)*

5. A 7×24 rectangle is inscribed in a circle. Find the radius of the circle.

   The face diagonal of a cube has length 4. Find the value of n given that $n\sqrt{2}$ is the *volume* of the cube. *(iTest)*

6. The difference between the areas of the circumcircle and incircle of an equilateral triangle is 300π square units. Find the number of units in the length of a side of the triangle.

7. Find the area of a triangle that has sides of length 5, 6, and 7.

8. Find the area of an equilateral triangle of side length 8.

9. Find the largest possible area of a triangle that has two sides of lengths 30 and 40.

10. Find the coordinates of the centroid of a triangle whose vertices have coordinates $(4, 7)$, $(-3, 4)$, and $(-7, 1)$.

11. Altitude CH and median BK are drawn in an acute triangle ABC, and it is known that $BK = CH$ and $\angle KBC = \angle HCB$. Prove that triangle ABC is equilateral. *(Leningrad Mathematical Olympiad)*

Number Sense

Many teachers erroneously dismiss number sense as a collection of worthless tricks. Poorly understood and taught, this might be the case. At MIST Academy we help students see the arithmetic, algebra, and number theory behind methods that make nearly all of computation, math, and problem solving easier.

1. Compute $65 \cdot 123 - 25 \cdot 123$.

2. Find the value of $\sqrt{72 \cdot 128 \cdot 75 \cdot 108}$.

3. Compute $2007^2 - 1993^2$.

4. Find the total number of digits in the product $8^{13} \cdot 25^{22}$.

5. What is the least natural number, greater than 1, that is a factor of $11000 + 1100 + 11$? *(MATHCOUNTS)*

6. Compute the value of A divided by B given that $A = 666666^6$ and $B = 222222^6$.

7. Find the GCD of 642 and 32172.

8. Compute $\frac{3^{2008} - 3^{2005}}{3^{2005} - 3^{2002}}$.

9. Compute the value of $50^2 - (100)(57) + 57^2$.

10. Compute $91^3 - 3 \cdot 91^2 + 3 \cdot 91 - 1$.

11. The six-digit decimal integer $349AB1$ is a perfect square, where A and B are decimal digits, not necessarily distinct. Find its three-digit square root.

Number Theory

Number theory, the mathematics of integers is one of the most unfortunately under-covered areas of mathematics, especially at the middle school level. In a world in which computing is largely based on logic and number theory, MIST Academy promotes number theory as a substantial part of its curriculum.

1. Find the value of $x + y$ where x and y are positive integers that satisfy

$$13x + 19y = 212.$$

2. What is the smallest prime factor of 1821? *(MATHCOUNTS)*

3. Convert each of the following to base 10:

(a)	71_8	(c)	$2A2_{11}$	(e)	2144_6
(b)	133_9	(d)	4040_8	(f)	1020102_3

4. A certain number n has factors of 15 and 10. What is the fewest total number of positive factors that n can have? *(MATHCOUNTS)*

5. In 1963, mathematicians at the University of Illinois used a computer to show that $2^{11213} - 1$ is a prime number. If this number were written in standard notation, it would contain 3376 digits. What would the units digit be? *(MATHCOUNTS)*

6. In how many ways can 16 rocks be divided into 5 piles such that the total number of rocks in each pile is odd?

7. A box contains gold coins. If the coins are equally divided among six people, four coins are left over. If the coins are equally divided among five people, three coins are left over. If the box holds the smallest number of coins that meets these two conditions, how many coins are left when equally divided among seven people?

   (A) 0 (B) 1 (C) 2 (D) 3 (E) 5

   (AMC 8)

8. Find $a + b$ given that the five-digit integer $3ab16$ is a perfect square.

9. For how many ordered pairs of digits (A, B) is $2AB8$ a multiple of 12? *(Vestavia ARML TST)*

10. Find the sum of the x-coordinates of all possible positive integral solutions to $\frac{1}{x} + \frac{1}{y} = \frac{1}{7}$. *(MATHCOUNTS)*

Probability

1. Jason rolls a pair of regular 6-sided dice.
   (a) Find the probability that the sum of the dots on the top of the two dice is 10.
   (b) Find the probability that the sum of the dots on the top of the two dice is *at least* 10.

2. Three numbers are randomly selected, with replacement, from the set of integers 1 through 100 inclusive. What is the probability that the product of the three numbers selected will be even? *(MATHCOUNTS)*

3. Jenny pulls two marbles out of a bag containing 4 red and 6 green marbles. Find the probability that the two marbles are of the same color.

4. Harold tosses a nickel four times. The probability that he gets at least as many heads as tails is
   (A) $\frac{5}{16}$ (B) $\frac{3}{8}$ (C) $\frac{1}{2}$ (D) $\frac{5}{8}$ (E) $\frac{11}{16}$
   (AMC 8)

5. Russell rolls a pair of dice with his eyes closed. His sister tells him that the sum of the dice is greater than or equal to nine. Knowing this information, Russell calculates the probability that he rolled doubles. What is that probability expressed as a common fraction? *(MATHCOUNTS)*

6. Instead of using two standard cubical dice while playing a board game, three dice are used so that the game goes more quickly. In the regular game, doubles (i.e., the same number rolled on both dice) are needed to get out of the pit. In the revised game, doubles or triples (i.e., the same number rolled on at least two of the three dice) are needed to get out of the pit. How many times as likely is it for a player to get out of the pit on one toss under the new rules as compared to the old rules? *(MATHCOUNTS)*

7. A fair six-sided die is rolled four times. Find the probability that at least two of the results are the same.

8. Amy and Martin play a game in which they take turn flipping a fair coin until one of them flips a head. The first person to flip a head is the winner. If Amy goes first, find the probability that she wins.

Sequences & Series

Problems involving sequences and series can challenge a student to apply master of arithmetic, pattern hunting, algebra, number theory, and sometimes other areas of mathematics. These problems often frustrate even talented solvers, but can be the most enjoyable at the same time.

1. Find the next three numbers in each of the following sequences:
   (a) 3, 6, 9, 12, 15, 18, __, __, __, ...
   (b) 4, 7, 10, 13, 16, 19, __, __, __, ...
   (c) 4, 8, 12, 16, 20, 24, __, __, __, ...
   (d) 3, 6, 12, 24, 48, 96, __, __, __, ...
   (e) 3, 7, 15, 31, 63, 127, __, __, __, ...

2. The first term of an arithmetic sequence is 13, and the seventh term is 31.
   (a) Find the common difference of the sequence.
   (b) Find a general formula for the sequence.
   (c) Find the twentieth term in the sequence.
   (d) If the last term in the sequence is 403, how many terms are in the sequence?

3. When the 171st even natural number is subtracted from the 219th odd natural number, the result is z. Find z. *(MATHCOUNTS)*

4. Big Al, the ape, ate 100 bananas from May 1 through May 5. Each day he ate six more bananas than on the previous day. How many bananas did Big Al eat on May 5?

   (A) 20 (B) 22 (C) 30 (D) 32 (E) 34

   (AMC 8)

5. What is the largest seven-digit number that contains each of the digits 1 through 7 and has the property that the sum of any two consecutive digits is a prime number? *(MATHCOUNTS)*

6. Look for a pattern:

$$\begin{aligned} 1^3 &= 1^2 - 0^2 \\ 2^3 &= 3^2 - 1^2 \\ 3^3 &= 6^2 - 3^2 \\ &\vdots \\ 6^3 &= n^2 - m^2 \end{aligned}$$

   What is the value of $m + n$? *(MATHCOUNTS)*

7. Consider the sum

$$7 + 77 + 777 + 7777 + \cdots + 77{,}777{,}777{,}777{,}777{,}777{,}777,$$

where the last addend has 20 digits. Find the digit in the hundreds place of the resulting sum. *(MATHCOUNTS)*

8. Compute the sum

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \cdots + \frac{1}{9900}.$$

9. Is it possible to write the natural numbers $1, 2, \ldots, 100$ in a row so that the difference between any two adjacent numbers is no less than 50? *(Leningrad Mathematical Olympiad)*