The 1st Annual iTest Tournament of Champions

Round 1 Problems

Answers to all problems are integers from 0 to 2007 inclusive. The use of calculators and computers is strictly prohibited. All answers should be submitted online within the iTest website according to the posted guidelines. Email tournamentofchampions@theitest.com with questions or for technical support.

1. (2 points) Find the remainder when 3^{2007} is divided by 2007.

2. (3 points) Let a/b be the probability that a randomly chosen positive divisor of 12^{2007} is also a divisor of 12^{2000}, where a and b are relatively prime positive integers. Find the remainder when $a+b$ is divided by 2007.

3. (5 points) For each positive integer n, let $g(n)$ be the sum of the digits when n is written in binary. For how many positive integers n, where $1 \leq n \leq 2007$, is $g(n) \geq 3$?

4. (7 points) Black and white coins are placed on some of the squares of a 418×418 grid. All black coins that are in the same row as any white coin(s) are removed. After that, all white coins that are in the same column as any black coin(s) are removed. If b is the number of black coins remaining and w is the number of remaining white coins, find the remainder when the maximum possible value of bw gets divided by 2007.

5. (8 points) Find the largest possible value of $a+b$, less than or equal to 2007, for which a and b are relatively prime, and such that there is some positive integer n for which

$$\frac{2^3-1}{2^3+1} \cdot \frac{3^3-1}{3^3+1} \cdot \frac{4^3-1}{4^3+1} \cdots \frac{n^3-1}{n^3+1} = \frac{a}{b}.$$