The 1st Annual iTest Tournament of Champions

Round 2 Problems

Answers to all problems are integers from 0 to 2007 inclusive. The use of calculators and computers is strictly prohibited. All answers should be submitted online within the iTest website according to the posted guidelines. Email tournamentofchampions@theitest.com with questions or for technical support.

1. (2 points) Let a and b be perfect squares whose product exceeds their sum by 4844. Compute the value of
$$(\sqrt{a}+1)\left(\sqrt{b}+1\right)(\sqrt{a}-1)\left(\sqrt{b}-1\right)-\left(\sqrt{68}+1\right)\left(\sqrt{63}+1\right)\left(\sqrt{63}-1\right)\left(\sqrt{68}-1\right).$$

2. (4 points) The area of triangle ABC is 2007. One of its sides has length 18, and the tangent of the angle opposite that side is 2007/24832. When the altitude is dropped to the side of length 18, it cuts that side into two segments. Find the sum of the squares of those two segments.

3. (5 points) Find the smallest value of n for which the series
$$1\cdot 3^1+2\cdot 3^2+3\cdot 3^3+\cdots+n\cdot 3^n$$
exceeds 3^{2007}.

4. (6 points) For each positive integer n, let $S_n=\sum_{k=1}^{n}k^3$, and let $d(n)$ be the number of positive divisors of n. For how many positive integers m, where $m\leq 25$, is there a solution n to the equation $d(S_n)=m$?

5. (8 points) A polynomial $p(x)$ of degree 1000 is such that $p(n)=(n+1)2^n$ for all nonnegative integers n such that $n\leq 1000$. Given that
$$p(1001)=a\cdot 2^b-c,$$
where a is an odd integer, and $0<c<2007$, find $c-(a+b)$.