

The 1st Annual iTest Tournament of Champions

Round 3 Problems

Answers to all problems are integers from 0 to 2007 inclusive. The use of calculators and computers is strictly prohibited. All answers should be submitted online within the iTest website according to the posted guidelines. Email tournamentofchampions@thetest.com with questions or for technical support.

- (3 points) Let A be the area of the locus of points z in the complex plane that satisfy $|z + 12 + 9i| \leq 15$. Compute $\lfloor A \rfloor$.
- (3 points) Al and Bill play a game involving a fair six-sided die. The die is rolled until either there is a number less than 5 rolled on consecutive tosses, or there is a number greater than 4 on consecutive tosses. Al wins if the last roll is a 5 or a 6. Bill wins if the last roll is a 2 or lower. Let m and n be relatively prime positive integers such that m/n is the probability that Bill wins. Find the value of $m + n$.
- (5 points) Find the largest natural number n such that

$$2^n + 2^{11} + 2^8$$

is a perfect square.

- (6 points) Find the smallest positive integer k such that

$$(16a^2 + 36b^2 + 81c^2)(81a^2 + 36b^2 + 16c^2) < k(a^2 + b^2 + c^2)^2,$$

for some ordered triple of positive integers (a, b, c) .

- (8 points) Let $s = a + b + c$, where a , b , and c are integers that are lengths of the sides of a box. The volume of the box is numerically equal to the sum of the lengths of the twelve edges of the box plus its surface area. Find the sum of the possible values of s .