

The 1st Annual iTest Tournament of Champions

Round 5 Problems

- (2 points) Find the smallest positive integer n such that a cube with sides of length n can be divided up into exactly 2007 smaller cubes, each of whose sides is of integer length.
- (4 points) In the game of *Winners Make Zeros*, a pair of positive integers (m, n) is written on a sheet of paper. Then the game begins, as the players make the following legal moves:
 - If $m \geq n$, the player chooses a positive integer c such that $m - cn \geq 0$, and replaces (m, n) with $(m - cn, n)$.
 - If $m < n$, the player chooses a positive integer c such that $n - cm \geq 0$, and replaces (m, n) with $(m, n - cm)$.

When m or n becomes 0, the game ends, and the last player to have moved is declared the winner. If m and n are originally 2007777 and 2007, find the largest choice the first player can make for c (on his first move) such that the first player has a winning strategy after that first move.

- (5 points) Find the sum of all integers n such that

$$n^4 + n^3 + n^2 + n + 1.$$

is a perfect square.

- (6 points) In triangle ABC , points A' , B' , and C' are chosen with A' on segment AB , B' on segment BC , and C' on segment CA so that triangle $A'B'C'$ is directly similar to ABC . The incenters of ABC and $A'B'C'$ are I and I' respectively. Lines BC , $A'C'$, and II' are all parallel. If $AB = 100$ and $AC = 120$, what is the length of BC ?
- (8 points) Let c be the number of ways to choose three vertices of an 6-dimensional cube that form an equilateral triangle. Find the remainder when c is divided by 2007.