The 1st Annual iTest Tournament of Champions

Round 6 Problems

1. (3 points) Given that

$$\begin{aligned} x &= 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots + \frac{1}{2007}, \\ y &= \frac{1}{1005} + \frac{1}{1006} + \frac{1}{1007} + \cdots + \frac{1}{2007}, \end{aligned}$$

find the value of k such that

$$x = y + \frac{1}{k}.$$

2. (4 points) Let

$$S = 1 + \frac{1}{8} + \frac{1 \cdot 5}{8 \cdot 16} + \frac{1 \cdot 5 \cdot 9}{8 \cdot 16 \cdot 24} + \cdots + \frac{1 \cdot 5 \cdot 9 \cdots (4k+1)}{8 \cdot 16 \cdot 24 \cdots (8k+8)} + \cdots .$$

Find the positive integer n such that $2^n < S^{2007} < 2^{n+1}$.

3. (5 points) Find the real number k such that a, b, c, and d are real numbers that satisfy the system of equations

$$\begin{aligned} abcd &= 2007, \\ a &= \sqrt{55 + \sqrt{k+a}}, \\ b &= \sqrt{55 - \sqrt{k+b}}, \\ c &= \sqrt{55 + \sqrt{k-c}}, \\ d &= \sqrt{55 - \sqrt{k-d}}. \end{aligned}$$

4. (6 points) Let $x_1, x_2, \ldots, x_{2007}$ be real numbers such that $-1 \le x_i \le 1$ for $1 \le i \le 2007$, and

$$\sum_{i=1}^{2007} x_i^3 = 0.$$

Find the maximum possible value of $\left\lfloor \sum_{i=1}^{2007} x_i \right\rfloor$.

5. (7 points) Acute triangle ABC has altitudes AD, BE, and CF. Point D is projected onto AB and AC to points D_c and D_b respectively. Likewise, E is projected to E_a on BC and E_c on AB, and F is projected to F_a on BC and F_b on AC. Lines D_bD_c, E_cE_a, F_aF_b bound a triangle of area T_1, and lines E_cF_b, D_bE_a, F_aD_c bound a triangle of area T_2. What is the smallest possible value of the ratio T_2/T_1?