

1. Two six-sided dice are constructed such that each face is equally likely to show up when rolled. The numbers on the faces of one of the dice are 1, 3, 4, 5, 6, and 8. The numbers on the faces of the other die are 1, 2, 2, 3, 3, and 4. Find the probability of rolling a sum of 9 with these two dice.
2. A large cube is painted green and then chopped up into 64 smaller congruent cubes. How many of the smaller cubes have at least one face painted green?
3. The difference between the areas of the circumcircle and incircle of an equilateral triangle is 300π square units. Find the number of units in the length of a side of the triangle.
4. For how many ordered pairs of digits (A, B) is $2AB8$ a multiple of 12?
5. A 2×2 square grid is constructed with four 1×1 squares. The square on the upper left is labeled A , the square on the upper right is labeled B , the square on the lower left is labeled C , and the square on the lower right is labeled D . The four squares are to be painted such that 2 are blue, 1 is red, and 1 is green. In how many ways can this be done?
6. How many of the positive divisors of 3,240,000 are perfect cubes?
7. Find the sum of the infinite series:

$$3 + \frac{11}{4} + \frac{9}{4} + \cdots + \frac{n^2 + 2n + 3}{2^n} + \cdots$$

8. Find the number of ordered pairs of integers (x, y) which satisfy

$$x^2 + 4x + y^2 = 21.$$

9. A square is inscribed in a circle of diameter 12. Two vertices of a triangle are also vertices of one side of the square. The other vertex of the triangle is on the circle. Find the largest possible area of the triangle.

10. When Jon Stewart walks up stairs he takes one or two steps at a time. His stepping sequence is not necessarily regular. He might step up one step, then two, then two again, then one, then one, and then two in order to climb up a total of 9 steps. In how many ways can Jon walk up a 14 step stairwell?
11. In concave hexagon $ABCDEF$, $\angle A = \angle B = \angle C = 90^\circ$, $\angle D = 100^\circ$, and $\angle F = 80^\circ$. Also, $CD = FA$, $AB = 7$, $BC = 10$, and $EF + DE = 12$. Compute the area of the hexagon.

12. Find the number of ordered pairs of positive integers (a, b, c, d) that satisfy the following equation:

$$a + b + c + d = 12.$$

13. There is one natural number with exactly 6 positive divisors, the sum of whose reciprocals is 2. Find that natural number.

14. Find the fourth smallest possible value of $x + y$ where x and y are positive integers that satisfy the following equation:

$$x^2 - 2y^2 = 1.$$

15. Virginia has a pair of fair 8 sided dice with faces numbered 1-8 on each die. Montana has a pair of fair 8 sided dice with faces numbered with positive integers in such a way that when her pair of dice is rolled, the probability of any particular sum occurring is the same as when Virginia rolls her dice. The largest number on any face of either of Montana's dice is 11. Find the sum of the numbers on the faces of the die whose faces are all less than 11.