Alabama ARML Team Selection Test
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1. How many integers x satisfy the inequality
$$(x-2006)(x-2004)(x-2002)\cdots(x-4)<0?$$

2. Compute $\sum_{k=1}^{5}\sum_{n=1}^{6} kn$.

3. River draws four cards from a standard 52 card deck of playing cards. Exactly 3 of them are 2's. Find the probability River drew exactly one spade and one club from the deck.

4. Find the number of six-digit positive integers for which the digits are in increasing order.

5. There exist positive integers A, B, C, and D with no common factor greater than 1, such that
$$A\log_{1200}2+B\log_{1200}3+C\log_{1200}5=D.$$
Find $A+B+C+D$.

6. Let $\lfloor a\rfloor$ be the greatest integer less than or equal to a and let $\{a\}=a-\lfloor a\rfloor$. Find $10(x+y+z)$ given that
$$x+\lfloor y\rfloor+\{z\}=14.2,$$
$$\lfloor x\rfloor+\{y\}+z=15.3,$$
$$\{x\}+y+\lfloor z\rfloor=16.1.$$

7. Four equilateral triangles are drawn such that each one shares a different side with a square of side length 10. None of the areas of the triangles overlap with the area of the square. The four vertices of the triangles that aren't vertices of the square are connected to form a larger square. Find the area of this larger square.

8. A bored mathematician has his computer calculate 1000 consecutive terms in the Fibonacci sequence. He notes that the smallest of the numbers is a multiple of 7. How many of the other 999 Fibonacci numbers are multiples of 7?

9. Amanda ordered a dozen donuts. She said she wanted only chocolate, glazed, and powdered donuts, and at least one of each kind. Let a, b, and c be the number of chocolate, glazed, and powdered donuts she wound up with. Find the number of possible ordered triples (a,b,c).

10. Let p be the probability that Scooby Doo solves any given mystery. The probability that Scooby Doo solves 1800 out of 2006 given mysteries is the same as the probability that he solves 1801 of them. Find the probability that Scooby Doo solves the mystery of why Eddie Murphy decided to stop being funny.

11. The integer 5^{2006} has 1403 digits, and 1 is its first digit (farthest to the left). For how many integers $0 \leq k \leq 2005$ does 5^k begin with the digit 1?

12. Yoda begins writing the positive integers starting from 1 and continuing consecutively as he writes. When he stops, he realizes that there is no set of 5 composite integers among the ones he wrote such that each pair of those 5 is relatively prime. What's the largest possible number Yoda could have stopped on?

13. Find the sum of the solutions to the equation

$$\sqrt[4]{x+27} + \sqrt[4]{55-x} = 4.$$

14. Find the real solution (x, y) to the system of equations

$$x^3 - 3xy^2 = -610,$$
$$3x^2y - y^3 = 182.$$

15. Ying lives on Strangeland, a tiny planet with 4 little cities that are each 100 miles apart from each other. One day, Ying begins driving from her home city of Viavesta to the city of Havennew, which takes her about an hour. When she gets to Havennew, she decides she wants to go straight to another city on Strangeland, so she randomly chooses one of the other three cities (possibly Viavesta), and starts driving there. Ying drives like this for most of the day, making 8 total trips between cities on Strangeland, choosing randomly where to drive to next from each stop. She then stops at her final city of destination, digs a hole, and buries her car.

    Let p be the probability Ying buried her car in Viavesta and let q be the probability she buried it in Havennew. Find the value of $p + q$.